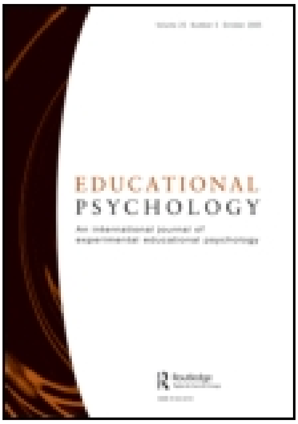

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In recent years, there has been increased interest in improving early mathematics curricula and instruction. Subsequently, there has also been a rise in demand for better early mathematics assessments, as most current measures are limited in their content and/or their sensitivity to detect differences in early mathematics development among young children. In this article, using data from two large samples of diverse populations of prekindergarten and kindergarten children, we provide evidence regarding the psychometric validity of a new theory-based early mathematics assessment. The new measure is the short form of a longer, validated measure. Our results suggest the short form assessment is valid for assessing prekindergarten and kindergarten children's numeracy and geometry skills and is sensitive to differences in early mathematics development among young children.

Keywords: early mathematics; prekindergarten; kindergarten; psychometrics; assessment

In recent years, there has been increasing interest in improving early mathematics curricula and assessment. For example, Head Start, the largest publicly funded preschool programme in the USA, is currently strengthening its mathematics curricula, while several states are implementing new early mathematics education programmes (Ginsburg, Lee, & Boyd, 2008). In explaining this spike in interest, early mathematics experts have pointed to several factors. First, the US children do not fare well in mathematics in international comparisons and gaps may begin in prekindergarten (Ginsburg et al., 2008; Sarama & Clements, 2009). Second, there is a well-developed body of evidence across disciplines suggesting that children begin developing mathematical competencies in infancy as part of the natural course of development (Butterworth, 1999; Clements & Sarama, 2007a, 2007c; Ginsburg, Cannon, Eisenband, & Pappas, 2006). This evidence has helped to dispel notions that children are not ready to learn mathematics in prekindergarten. Third, some research suggests that mathematics ability at kindergarten entry is a strong predictor of later academic outcomes, stronger even than early reading ability (Duncan et al., 2007). Finally, there is increasing concern and awareness regarding gaps in mathematics

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achievement between children from low-resource families and their more advantaged peers, with gaps particularly prevalent among some subgroups, such as students who are black, Hispanic or English Language Learners (Clements, Sarama, & Liu, 2008; Ginsburg et al., 2008; National Center for Education Statistics, 2000).

The increasing focus on early mathematics and demand for evidence-based early mathematics curricula have also created a demand for better early mathematics assessments. Clements et al. (2008) review several commonly used early mathematics instruments and point out that these measures are generally limited in their content and/or their sensitivity to detect differences in early mathematics development among young children. The research-based early mathematics assessment (REMA), developed by Clements, Sarama and Liu, is designed to address these gaps. The REMA is based on theory and empirical work regarding learning trajectories in mathematics learning (Clements, 2007) and taps a broader range of early mathematics skills than more widely used measures. These learning trajectories are defined as 'descriptions of children's thinking and learning in a specific mathematical domain, . . ., created with the intent of supporting children's achievement of specific goals in that mathematical domain' (Clements & Sarama, 2004b, p. 83).

Within the REMA, learning trajectories are conceptualised within two broad domains – number and geometric/spatial competencies. Number constructs include verbal and object counting, subitising, number comparison and sequencing, connecting numerals to quantities, number composition/decomposition, adding and subtracting and place value. Geometric/spatial constructs include the recognition, composition/decomposition, congruence and construction of shapes, as well as spatial imagery, measurement and patterning.

Using three different pilot phases and extensive Rasch analysis, Clements et al. (2008) established that the REMA has excellent psychometric properties, including good construct validity. The REMA also showed good concurrent validity with other mathematics assessments in two different studies: one study found a .86 correlation between the REMA and another complete measure of preschool mathematics achievement (Klein, Starkey, & Wakeley, 2000), while the other found correlations between .62 and .76 between REMA and Woodcock-Johnson III Applied Problems at three different time points (end of prekindergarten, end of kindergarten and end of first grade; Sarama, Clements, Wolfe, & Spitler, submitted for publication). The REMA forms the foundation for the recently published tools for early assessment of mathematics (TEAM; Sarama, Clements, & Wolfe, 2010), a formative assessment designed for prekindergarten through fifth grade.

In this article, we extend the work of Clements et al. (2008) by validating a 19-item Short Form version of the full 125-item REMA. This Short Form version includes items designed to tap into only the earliest learning trajectories related to mathematical development and can be used to assess prekindergarten and kindergarten children's numeracy and geometry skills. Although the Short Form measure is limited in its use with older children, it is nonetheless a useful tool for studies of young children and in early care settings as a formative assessment of child deficits and strengths in early mathematics. In the present study, we administered items to two independent samples of prekindergarten and kindergarten children to examine the properties of the Short Form. We found it is sensitive to ability differences in young children's early mathematics skills and has strong validity.

Need for new early mathematics measures

The field of early childhood education has a generous supply of achievement measures, but most assess literacy. The mathematics assessments available are often either lengthy (including an unsystematic amalgamation of different instruments) or short but inadequate. For example, a commonly used instrument is the Woodcock-Johnson III (Woodcock, McGrew, & Mather, 2001), especially the Applied Problems subtest. This subtest has several strengths, including assessing a wide range of abilities and ages, reliabilities above .80, and large normative data samples. However, two national panels on preschool assessment (National Institute of Child Health & Human Development Forum, Washington, DC, June 2002; Center for Improving Resources in Children's Lives Forum, Temple University, 30–31 January 2003) cautioned against the sole use of the Woodcock-Johnson for assessment of mathematical skills in preschoolers because it covers a narrow range of problems (e.g. it has multiple tasks in which children must count a proper subset, all with numbers from 1 to 4) and jumps too quickly to advanced, formal knowledge. In addition, it is not based on current research of the development of mathematical thinking, paying little attention to developmental sequences. Other assessments have similar limitations. For example, the Bracken Basic Concept Scale (Bracken, 1984/1998) includes several mathematical concept categories; however, the national panels cautioned content validity was low (including mathematically questionable items such as one-dimensional 'linear shapes'). Further, because Bracken subtests are not intended to be used separately, it can be difficult to administer or interpret results for mathematical topics.

A measure more positively reviewed by the national panels is the test of early mathematics ability (Ginsburg & Baroody, 2003). However, although the official description of the instrument states that it 'measures the mathematics performance of children', this is only partially true. It measures number competencies, but not some other important mathematical skills. Even assessments that include content other than number lack substantial coverage of geometry and measurement skills (Chernoff, Flanagan, McPhee, & Park, 2007).

Recognising the critical need for improving and expanding early math education, a recent National Research Council Report (2009) called for better and more comprehensive measures. The REMA was developed to meet that call. However, it grew to involve two administration sessions, often with durations of greater than 30 min each and discouraged its use in favour of shorter, albeit inadequate, instruments. The challenge, therefore, was to build a short form of the REMA that would correlate highly with the existing instrument.

Learning trajectories' developmental progressions

A key strength of the REMA and its Short Form is that they are based on learning trajectories. Learning trajectories (also known as developmental progressions) have played a substantial role in recent theory and research (Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009; Wilson, 2009), as well in as the development of standards and curriculum (Daro et al., 2011). The ongoing work of *The Common Core State Standards Initiative*, a joint effort by the National Governors Association Center for Best Practices and the Council of Chief State School

Officers, is based to a large extent on such developmental progressions/learning trajectories.

To give an example of a learning trajectory, we turn to the shape composition trajectory. The developmental progression for this skill was based on observations of children's explorations (Sarama, Clements, & Vukelic, 1996), corroborated by data from other researchers (Mansfield & Scott, 1990; Sales, 1994; Vurpillot, 1976), and refined through a series of clinical interviews, focused observations (Clements, Sarama, & Wilson, 2001) and statistical analyses (Clements, Wilson, & Sarama, 2004). Children gain abilities in this domain first by learning to compose individual geometric shapes and then learning to combine them – initially through trial and error and gradually by attributes – into pictures, and finally synthesise combinations of shapes into new shapes (composite shapes). The REMA Short Form includes a series of items that tracks this developmental progression.

Research questions

In the present study, we examine the psychometric properties of the REMA Short Form. We address six research questions:

(1) Does a shortened set of mathematical items for 4- and 5-year-old children taken from a longer, validated measure fit the Rasch model?
(2) Does the Short Form differentiate students of differing ability levels?
(3) Are Short Form items robust across two independent samples of 4- and 5-year-old children?
(4) Does the Short Form demonstrate strong concurrent validity with the Woodcock-Johnson Applied Problems subscale and the full REMA, and strong discriminant validity with cognitive tests in non-mathematics domains?
(5) Do the Short Form items show differential item functioning (DIF) across subgroups of students (home language, free/reduced lunch status, gender and grade level)?
(6) Can a stop rule be used with the Short Form without substantial loss of information?

In brief, we hypothesised that the answer to all research questions was 'yes', except for research question (5) where we hypothesised that the measure would not show DIF across subgroups of students.

Method

Sample

Sample 1 for the Short Form included 1930 prekindergarten ($N = 1002$) and kindergarten ($N = 928$) children in a large urban public school district in the north-eastern USA. Sample 1 children were tested in fall 2009 as part of a larger regression–discontinuity evaluation of the district's 4-year-old prekindergarten programme. All children in Sample 1 were either enrolled in the district's prekindergarten programme at the time of assessment or had attended the programme in the previous year. Prekindergarten classrooms in the district used the *Building Blocks* early mathematics curriculum (Clements & Sarama, 2007b), as well as the literacy curriculum *Opening the World of Learning*, which is designed for full-day implementation, with components added to language and literacy, including mathematics, science, social studies, art and physical, social and emotional development (Schickedanz & Dickinson, 2005). As both curricula were developed for use with preschool-aged children, the district used other, different curricula in the kindergarten programme (specifically, *Investigations* for mathematics and *Reading Street* for literacy). Sample children were drawn from 67, or 85%, of the district's elementary schools.

Sample 2 was drawn from a randomised controlled trial of the *Building Blocks* curriculum in two large urban school districts in the north-eastern USA beginning in the fall of 2006.[1] This sample includes 1305 prekindergarten children tested prior to prekindergarten mathematics instruction and at the end of the prekindergarten year. In total, 93% of the sample was also tested at the end of kindergarten ($N = 1218$). The two districts were chosen because they had ethnically diverse populations, they served primarily children from low-resource communities, and students generally remained in the same schools for prekindergarten and kindergarten. Within each district, 42 participating schools were randomly assigned to one of three curricular conditions: business-as-usual, *Building Blocks* or *Building Blocks* enhanced condition.[2] In the prekindergarten year, the *Building Blocks* and *Building Blocks* enhanced conditions were identical. In the kindergarten year, teachers in the enhanced condition received additional training to support children as they moved from prekindergarten. Specifically, these kindergarten teachers were provided access to the *Building Blocks* software suite as well as professional development focused on how to adapt their core curriculum (*Investigations*) to take advantage of the increased mathematics competencies of their incoming kindergarten with prior *Building Blocks* experience. Within each prekindergarten classroom, up to 15 children were randomly selected for testing. Within the business-as-usual conditions, teachers utilised the same base curriculum of *Investigations* but did not receive any professional development in mathematics during the intervention. Sample 1 children received the Short Form, while Sample 2 children received the full REMA.

Table 1 provides details on the demographics of the samples. Sample 1 had proportionally fewer blacks and more Hispanic children than Sample 2. Sample 2 also had a higher proportion of children eligible for free/reduced lunch than Sample 1 (85 vs. 69%, respectively).

Table 1. Sample descriptive statistics.

Sample characteristic	Sample 1 ($N = 1929$) proportion	Sample 2 ($N = 1305$) proportion
Black	.27	.54
White	.19	.19
Hispanic	.41	.22
Asian	.11	.04
Other race/ethnicity	.03	.01
Free/reduced lunch eligible	.69	.85
Male	.51	.49

Note: One Sample 1 child was missing demographic information.

Development of the Short Form and content validity

Clements et al. (2008) detail the process of developing the full REMA; for brevity, we did not include full details here. In brief, their process article includes how the REMA's content goals were determined and describes the three pilot tests used to refine test items (Clements & Sarama, 2004a, 2007a, 2007c; Sarama & Clements, 2002). They also present and discuss results from submitting the measure to a Rasch model, with 'mathematical competence' as the defined latent trait. Their choice of a Rasch model allowed them to locate children on a common ability scale with a consistent, justifiable metric (Bond & Fox, 2007).

Items for the Short Form given to Sample 1 were selected from the full REMA item bank. Item selection prioritised those items with adequate fit to the Rasch model and that represented the full range of early mathematics competencies applicable within the prekindergarten and kindergarten periods of mathematics development (see Tables 2 and 3 for a description of the 19 items selected, including the specific competency tapped by each item). Items selected represent the competencies and learning trajectories of mathematics learning that are the most essential in prekindergarten and kindergarten children. They test the earliest levels within those number competencies considered fundamental mathematically, psychologically and educationally (e.g. CCSSO/NGA, 2010; Geary, 2006; Sarama & Clements, 2009), including recognition of number and subitising (four items),[3] composition of number (early arithmetic combinations, two items), comparing number and sequencing (two items) and counting (both verbal and object, two items). Basic numeral identification (one item) and arithmetic (addition; one item) were also included to capture the highest and lowest skill levels within the target population. Also included were items assessing geometry competencies, including the earliest

Table 2. Item descriptive statistics.

	Sample 1						Sample 2					
Item	*N*	Mean	Standard deviation	Min	Max	PB	*N*	Mean	Standard deviation	Min	Max	PB
1	1930	.94	.23	0	1	.34	2122	.94	.24	0	1	.32
2	1930	.96	.20	0	1	.27	2701	.89	.31	0	1	.51
3	1930	.85	.36	0	1	.49	2342	.77	.42	0	1	.60
4	1930	.26	.44	0	1	.51	1122	.43	.49	0	1	.46
5	1927	.06	.23	0	1	.34	1070	.18	.38	0	1	.51
6	1929	.74	.40	0	1	.66	2164	.81	.35	0	1	.51
7	1930	.79	.41	0	1	.44	2321	.71	.46	0	1	.44
8	1927	.49	.50	0	1	.58	2047	.48	.50	0	1	.57
9	1930	.32	.47	0	1	.46	1402	.53	.50	0	1	.50
10	1930	.08	.28	0	1	.42	777	.14	.34	0	1	.46
11	1929	.35	.48	0	1	.59						
12	1925	.27	.44	0	1	.35	1164	.41	.49	0	1	.45
13	1929	2.71	.49	0	3	.39	3571	2.41	.58	0	3	.47
14	1929	1.55	.74	0	3	.35	2741	2.08	.71	0	3	.46
15	1930	1.26	.85	0	2	.61	2711	.97	.82	0	2	.67
16	1929	.50	.50	0	1	.48	2281	.47	.50	0	1	.43
17	1930	.39	.49	0	1	.50	101	.80	.40	0	1	.37
18	1928	.15	.35	0	1	.43						
19	1929	.21	.41	0	1	.32	139	.38	.49	0	1	.52

Note: PB denotes point-biserial correlations between each item and the total raw score.

Table 3. Item descriptions, core competencies assessed and topic.

Item	Description	Core competency	Level of thinking in the learning trajectory (Clements & Sarama, 2009)
1	Count to five	Counting – verbal	Reciter (counts to five); Reciter (10) – counts to 10–20; Counter to 100 – counts 21 and higher
2	Compare quantities (3 and 4) and identify larger quantity	Comparing number and sequencing	Non-verbal comparer of similar items
3	Subitise 3	Recognition of number and subitising	Perceptual subitiser to 4
4	Subitise 10	Recognition of number and subitising	Conceptual subitiser to 10
5	Subitise 15	Recognition of number and subitising	Conceptual subitiser to 20
6	Match numeral to set – 5	Numerals	Numerals
7	Count eight objects	Counting – object	Corresponder
8	Count four objects	Recognition of number and subitising	Producer (small numbers)
9	Hide 3, show 2, how many?	Composition of number	Composer to 4, then 5
10	Hide 4, show 6, how many?	Composition of number	Composer to 10
11	Add 7 and 5	Arithmetic (adding, subtracting, multiplying, dividing)	Make it $N\pm$
12	Which is smaller: 27 or 32?	Comparing number and sequencing	Place value comparer
13	Identify triangles	Shape	Shape recogniser
14	Identify rhombuses	Shape	Identifying shape
15	Use straws to make a triangle	Shape	Constructor of shapes from parts – looks like for partially correct; Constructor of shapes from parts
16	Identify sides of a geometric shape	Shape	Side recogniser
17	Make ABB pattern	Patterning	Pattern duplicator
18	Identify rectangle	Shape	Constructor shapes from parts – exact
19	Identify triangle and trapezoid	Compose shape	Shape decomposer (with help)

trajectory levels within shape (five items) and shape composition (one item). Finally, an additional question at the lowest level of patterning was also included.

Other criteria for item selection were content, item duration and item materials. Items were selected from content areas thought to act as building blocks for later mathematics development and that are readily observable in young children (Sarama & Clements, 2009). To keep the Short Form brief and easy to administer, when two items showed approximately equal fit to the Rasch model, had similar difficulties and tapped the same early mathematics trajectory, we prioritised the item with few to no manipulative materials that required a shorter amount of administration time over longer items or those with multiple manipulatives.

In summary, the Short Form is based on both theory (Sarama & Clements, 2009), empirical evidence regarding the development of mathematics competencies in the prekindergarten and kindergarten periods (Clements et al., 2008, 2011), practical considerations and considerable pilot testing and psychometric analysis (Clements et al., 2008). As such, we believe that the Short Form has strong content validity and is practical for use in classroom settings and in cross-sectional research studies.

Procedures

Sample 1 children were tested by study-trained child assessors. Assessors had to prove reliability on a full battery of tests, including the REMA Short Form, and show good rapport/child management skills in both simulated and real testing situations. All assessors were college-educated, and approximately one-third held masters, degrees. On average, the Short Form took approximately 15–20 min to administer. Children were administered all 19 items unless the child did not speak English well enough to continue, had a severe disability that interfered with the testing session or displayed a behaviour problem that required the discontinuation of the testing session ($N = 30$ children). All children included in the sample were able to complete at least 14 of the 19 items.

Sample 1 children were also tested on the Woodcock-Johnson Applied Problems subscale, the Woodcock-Johnson Letter–Word Identification subscale and the Peabody Picture Vocabulary Test-III (PPVT-III).[4] The Applied Problems subscale (Woodcock et al., 2001) is a numeracy and early mathematics measure that requires children to perform relatively simple calculations to analyse and solve arithmetic problems. Its test–retest reliability for 2–7-year-old children is .90 (Woodcock et al., 2001) and it has been widely used with diverse populations of young children (Gormley, Gayer, Phillips, & Dawson, 2005; Howes et al., 2008; Peisner-Feinberg et al., 2001; Wong, Cook, Barnett, & Jung, 2008). The Woodcock-Johnson Letter-Word Identification subscale (Woodcock et al., 2001) is likewise a nationally normed, widely used measure (Gormley et al., 2005; Peisner-Feinberg et al., 2001). The Letter-Word Identification subscale asks children to identify and pronounce isolated letters and words fluently. According to the developers, test–retest reliability of the Letter-Word subscale for 2–7-year-olds is .96. Children's receptive vocabulary was measured using the PPVT-III (Dunn & Dunn, 1997), a nationally normed measure that has been widely used in diverse samples of young children (Love et al., 2005; US Department of Health & Human Services, 2010; Wong et al., 2008). The test has excellent split-half and test–retest reliability, as well as strong qualitative and quantitative validity properties. It requires children to choose verbally or non-verbally which of the four pictures best represents a stimulus word.

In Sample 2, the full REMA was administered by masters' level teachers across two videotaped testing sessions, each lasting 30–45 min. Assessors had to meet 98–100% reliability, determined by exact agreement between in-house coding team and individual assessor on administration across four assessments prior to actual data collection in sessions with volunteer children outside of the study. Based on the pilot testing and resultant Rasch model, items were arranged by level of difficulty and testing was discontinued after a child answered four consecutive items incorrectly.

Seventeen of the 19 items included on the Short Form used with Sample 1 were also used in the full REMA assessment given to prekindergarten and kindergarten students in Sample 2. There were two items included on the Short Form that were not administered to Sample 2 prekindergarten and kindergarten students. Sample 2 prekindergarten and kindergarten children were not tested on these two items due to timing; these two items were not piloted until this sample of children were in first grade. The two items were included on the Short Form for content validity purposes.

Item scoring

Fifteen of the 19 items on the Short Form were scored dichotomously. One number item (item 6) required children to match five numeral cards to the correct cards with that number of objects. Each correct match was scored +.2 for a total possible score of 1. One geometry item (item 15) involved giving children about eight short straws of different lengths and asking them to make a triangle using the straws. A triangle with three sides with no gaps or overlapping lines was scored as 2; a triangle with gaps and/or overlapping lines was scored as 1. Two geometry items – items 13 and 14 – involved asking children to choose all the triangles or rhombuses out of a set of 26 shapes, respectively. Each shape was an item, weighted as follows (with examples for triangles): palpable distractors (those without any overall resemblance such as ovals), .05; exemplars (theoretically- and empirically-determined prototypes of the class, such as an equilateral triangle), .25; variants (other members of the class, such as obtuse triangles), .50 and distractors (non-examples that are easily confused with examples, such as a triangular shape missing one of the three vertices), also .50.

Analysis

As with other studies that have used the REMA instrument (Clements et al., 2008), we used the Rasch model to obtain both the item difficulty (D_i) and the person ability estimates (B_n). Georg Rasch, the eponymous developer of the Rasch model, introduced the concept of placing these two metrics on the same scale (Rasch, 1980), allowing one to estimate the probability of answering an item correctly conditional on the Rasch-estimated ability score of the individual. Masters (1982) advanced Rasch's model by allowing for 'partial credit' in which the outcome is no longer dichotomous but polychotomous. Equation (1) represents the mathematical formula used in the Rasch partial credit model, and it expresses the probability that person n scored a value of x on item i. This equation is adapted from Cheng, Chen, Liu, and Chen (2011), and incorporates an ability parameter, B, an item difficulty parameter, D and an additional parameter, δ that expresses a change in probability of scoring $j+1$, rather than j on item i

$$P_{nix} = \frac{\exp \sum_{j=0}^{x} [B_n - (D_i + \delta_{ij})]}{\sum_{k}^{m_i} \exp \sum_{j=0}^{k} [B_n - (D_i + \delta_{ij})]}, x = 0 \cdots m_i. \tag{1}$$

For the dichotomous items, Equation (1) is reduced to Equation (2) (Bond & Fox, 2007). The elegance of these formulae is that the person's ability and item difficulty values are placed on the same scale. For example, from Equation (2), an individual with an ability of 2 logits has a 50% chance of answering correctly an item with a difficulty of 2 logits

$$P_{nix} = \frac{\exp(x(B_n - D_i))}{1 + \exp(B_n - D_i)}, \quad x = 0, 1. \tag{2}$$

The joint maximum likelihood estimation of both the person's ability and item difficulty parameters in the Rasch model provides another distinct advantage over simply assigning a person's ability as the percent of items answered correctly. Differences in difficulties across test items suggest that the gap in ability levels between an individual who answered eight items correctly and one who answered nine items correctly is not identical to the gap between an individual who answered 14 items correctly and one who answered 15 items correctly. By having different spacing between Rasch-based scores, we better reflect the true ability differences between people. Using raw scores, rather than the Rasch-estimated ability levels as an outcome measure, might obscure the true impact of any study using the REMA or similar instruments to generate outcomes.

The Rasch approach has several other advantages over raw scores as well. It allows us to place item difficulty and child proficiency on the same scale, which is both convenient and easy to interpret. We also chose the Rasch approach because it helps us to evaluate whether the underlying construct of early mathematics competency is uni-dimensional. This is an important advantage over using raw scores, given that a major contribution of the REMA is its inclusion of a broader range of early mathematic competencies when compared with the more commonly used Woodcock-Johnson Applied Problems subscale. Use of a raw score would make an implicit assumption of uni-dimensionality, whereas Rasch statistics allow us to evaluate the tenability of that assumption. Finally, using a Rasch model best reflects how the Short Form items were selected from the full item bank (i.e. they were selected in part because in full REMA analysis, they showed adequate fit to the Rasch model).[5]

Results

Descriptive statistics

Item descriptive statistics: Sample 1

As shown in Table 2, items differed in their mean scores, ranging from 96% of Sample 1 children getting an item correct (item 2) to just 6% of Sample 1 children getting an item correct (item 5). The average score on the multi-part numeral matching item (item 6) was .74 out of a possible score of 1. On the triangle construction item (item 15), the average score was 1.26 out of a possible score of 2. The average score on the multiple category triangle item (item 13) was 2.71 out of a possible score of 3 and the average score on the multiple category rhombus item (item 14) was 1.55 out of a possible score of 3.

Item descriptive statistics: Sample 2

Table 2 also displays the mean scores for the measured items included on the Short Form for Sample 2. As a result of a stop rule in place during administration, the number of children taking each item varies in Sample 2. The range of mean scores for children getting an item correct spans from 94% (item 1) to 14% (item 10). The

Table 4. Item difficulty, standard error, and fit (infit and outfit, mean square residual and standardised) statistics.

	Sample 1						Sample 2					
			Infit		Outfit				Infit		Outfit	
Item	Difficulty	Error	Mean-square fit statistics (MNSQ)	ZSTD	MNSQ	ZSTD	Difficulty	Error	MNSQ	ZSTD	MNSQ	ZTSD
1	−3.58	.11	.90	−1.25	.64	−2.40	−3.69	.10	.98	−.29	.80	−1.39
2	−3.99	.12	.95	−0.51	.86	−.71	−3.67	.08	.96	−.67	.93	−.56
3	−2.32	.07	.85	−3.64	.64	−4.01	−2.19	.06	.82	−6.23	.66	−6.29
4	1.28	.06	.90	−3.65	.83	−2.89	1.56	.07	.96	−1.60	.95	−1.21
5	3.43	.11	.90	−1.27	.57	−2.94	3.12	.09	.86	−2.73	.77	−2.76
6	−1.06	.06	.73	−9.90	.61	−8.88	−1.67	.06	.83	−4.88	.67	−5.38
7	−1.79	.06	.96	−1.18	1.10	1.33	−1.53	.05	1.03	1.27	1.02	.48
8	−.02	.05	.87	−6.36	.82	−5.08	−.15	.05	.86	−7.38	.83	−5.60
9	.92	.06	.99	−.30	1.03	.61	.97	.06	.91	−4.21	.88	−3.47
10	2.92	.09	.83	−2.77	.55	−3.78	3.77	.11	.87	−1.75	.82	−1.49
11	.73	.05	.84	−6.97	.74	−6.16						
12	1.22	.06	1.13	4.34	1.27	4.12	1.68	.07	.97	−.99	.96	−.99
13	−2.58	.04	1.22	5.59	1.35	6.33	−2.31	.03	1.29	9.90	1.44	9.90
14	1.30	.04	1.46	9.90	1.46	9.90	−.41	.03	1.17	4.54	1.20	4.90
15	−.69	.04	1.05	1.48	1.15	2.57	.63	.03	.86	−6.19	.83	−5.96
16	−.11	.05	1.03	1.20	1.06	1.66	.90	.05	1.02	1.14	1.07	2.29
17	.53	.05	.97	−1.45	1.02	.37	.37	.28	1.03	.27	.97	.02
18	2.18	.07	.94	−1.31	.81	−1.95	.63	.03	.86	−6.19	.83	−5.96
19	1.64	.06	1.13	3.54	1.34	4.13	2.61	.20	.97	−.34	.88	−.82

average score on the multi-part numeral matching item (6) was .81 out of a possible score of 1. The average score for the triangle construction question was .97 out of a possible two points. The average score on the multiple category triangle item (13) was 2.41 out of a possible three points and the average score on the multiple category rhombus item (item 14) was 2.08 out of a possible three points.

Overall, the probability of getting a particular item correct was similar across the two samples, although there were some differences. For example, for item 17, the percent correct was 39% for Sample 1 and 80% for Sample 2. Cronbach's alpha was also similar across the two tests (Sample 1, standardised = .79; Sample 2, standardised = .71), indicating reasonable internal consistency. The point-biserial correlations between item scores and the total raw score for both samples are shown in Table 2. Most were similar in magnitude across both samples. Differences in the strength of the correlations between individual questions and total test scores were noted between the samples for item 2, 5, 17 and 19 (all but 17 were stronger in Sample 2, while item 17 was stronger in Sample 1).

RQ 1: Do the Short Form items adequately fit the Rasch model?

To address research question 1, we submitted data from sample children to a Rasch model, using the Winsteps software (Linacre, 2005). Because no single fit statistic is sufficient to indicate item misfit, we examined several fit statistics, reliabilities and separation indices. As we explain below, the items adequately fit the Rasch model.

We examined both infit and outfit statistics; both are measures of the mean square error, but infit gives more weight to respondents with ability levels close to an item's difficulty, whereas outfit is not weighted. All items in both samples had infit and outfit statistics under or very close to the benchmark of 1.3 for determining adequate fit to the Rasch model (Bond & Fox, 2007).[6] As shown in Table 4 and in Sample 1, two items – 13 and 19 – had outfit statistics that were just over the 1.3 benchmark but had adequate infit statistics. In Sample 2, item 13 had an outfit statistic slightly larger than the 1.3 benchmark (1.44) but had an adequate infit statistic. In Sample 1, one item – item 14 – had both an outfit and an infit statistic of 1.46. We retained this item, as its infit and outfit values were relatively close to the benchmark and as its fit statistics were adequate in Sample 2.[7]

We also examined item reliability and person reliability statistics. Item reliability was approximately 1.00, suggesting that the difficulty rank order of this study's items would be constant or close to constant across different samples drawn from the population to which we generalise. Person reliability was .76. Within Sample 2, item reliability was also approximately 1.00, while the person reliability was .68. There are no explicit benchmarks for acceptable item and person reliability statistics that we are aware of but we interpret the ones reported here as adequate.

RQ 2: Does the Short Form provide information about students of differing ability levels?

Generally, tests should incorporate a range of item difficulties in order to obtain reliable information about children of different ability levels. As shown in Table 3, the difficulty of the 19 items in Sample 1 ranged from −3.99 logits (easiest) to 3.43 logits (hardest). In Sample 2, the range was −3.69 logits (easiest) to 3.77

(hardest). In terms of distribution within Sample 1, four items were very easy (difficulty less than −2 logits), five fell in the easy to average range (difficulty greater than −2 logits and less than 0 logits), seven items fell in the average to hard range (difficulty greater than 0 logits but less than 2 logits) and three items were very hard (difficulty greater than 2 logits). Within Sample 2, the distribution of item difficulty was identical, except that there were three which fall in the easy to average range (difficulty greater than −2 logits and less than 0 logits) rather than five, due to the fact that two items were not given to Sample 2.

Notably, the items had different difficulty spacings, or distances in difficulty between items when items are ordered by difficulty. By definition, any test does a better job discriminating between students whose ability scores fall between more tightly spaced items than those whose ability scores fall between more loosely spaced items. Nonetheless, the unequal spacing of the Short Form test items is acceptable, given that the test is not high stakes and given its purpose as a tool for cross-sectional studies of young children and as a formative assessment of child deficits and strengths in early mathematics. Those needing more finely tuned child-level ability estimates are encouraged to use the full REMA.

Figures 1 and 2 – histograms of the estimated ability scores for children in Samples 1 and 2, respectively – provide further evidence that the Short Form is well targeted to provide information about students of different ability levels, given its intended purpose. The distribution of ability scores appears to be distributed normally, with the majority of scores spread over a fairly wide range (from −3 to + 3).

RQ 3: Are Short Form items robust to differences in sampling?

A necessary condition of the Rasch model is that the item parameters are sample independent (see Andrich, 1988). In the context of this study, the implication is that

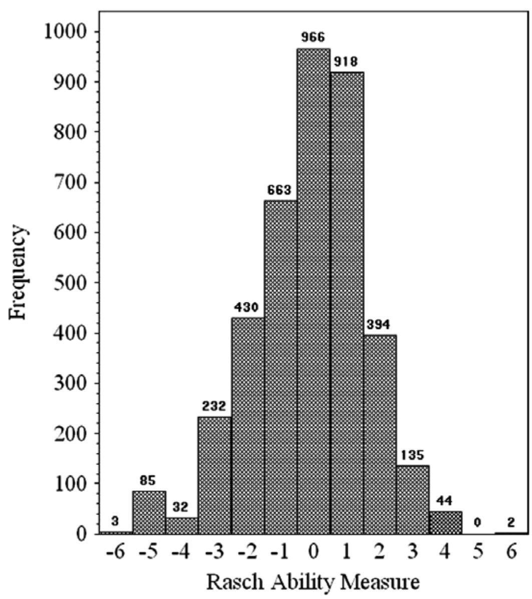

Figure 1. Histogram of Rasch-estimated ability, Sample 1.

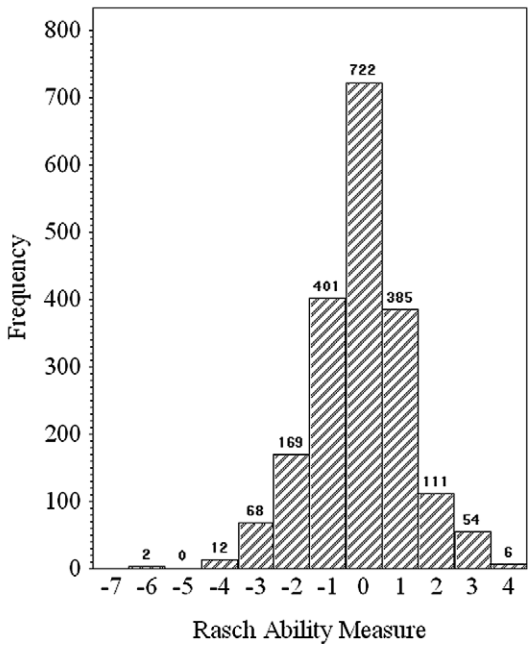

Figure 2. Histogram of Rasch-estimated ability, Sample 2.

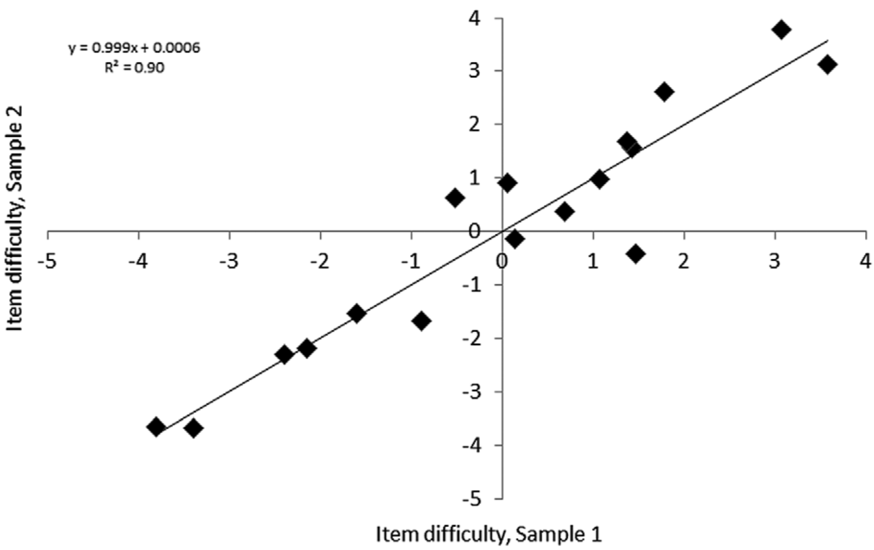

Figure 3. Scatterplot of Rasch-estimated difficulty of items given in both Sample 1 and Sample 2.

item parameter estimates obtained from the REMA administration should be identical across any sample drawn from the general population to which we are generalising. In Figure 3, we plot the estimated item difficulties of the 17 items given to prekindergarten and kindergarten students in both Sample 1 and Sample 2.

We also show a best-fit linear regression line of the relationship between the item difficulties in the two samples. The correlation between item difficulty estimates is very high across the two samples ($r = .95$), suggesting that the item parameter estimates are indeed sample independent. We find these results particularly encouraging, given that assessors were trained by different staff, and test items were administered to the samples in different years.

RQ 4: Does the Short Form demonstrate strong concurrent validity with the Woodcock-Johnson Applied Problems subscale and the full REMA and strong discriminant validity with non-mathematics cognitive tests?

Concurrent validity

Concurrent validity is determined by how well an assessment correlates to some other measure of the same, or similar, latent characteristic (Bond & Fox, 2007). A strong correlation between a new assessment and an older, more established assessment provides additional support for the new assessment's validity. In our work, within Sample 2, we assessed the relationship between the Short Form and the longer full form of the REMA. Using two time points – the beginning of prekindergarten and the end of prekindergarten– we correlated children's Rasch-estimated scores from the Short Form with their Rasch-estimated scores from the full REMA. In bivariate correlations, the Short Form scores evinced strong correlations with full REMA scores within each time point: at the beginning of prekindergarten, .71 ($p < .001$) and at the end of prekindergarten, .74 ($p < .001$). These results suggested that the Short Form has good concurrent validity with full REMA.

As a second concurrent validity check, we also correlated Rasch-estimated child ability scores on the Short Form to child raw scores on the Woodcock-Johnson Applied Problems subscale (Woodcock et al., 2001). The latter test was given to children only in Sample 1, in the same testing session as the Short Form test. We expected a moderate correlation between the two tests, given that the Short Form taps early mathematics skills that Applied Problems does not (particularly geometry) and that Applied Problems jumps more quickly to advanced, formal knowledge (Clements et al., 2008). Sample 1 children averaged a raw score of 13.80 on the Woodcock-Johnson Applied Problems subscale (standard deviation of 5.24). The correlation between children's Applied Problems and Short Form Rasch-estimated ability scores was .74 ($p < .001$).

Discriminant validity

Cognitive tests tend to be moderately to highly correlated, even across domains. To examine discriminant validity – or the degree to which the Short Form is measuring something other than general cognitive ability – we also correlated our Rasch-estimated child ability scores to two other cognitive but non-mathematics tests given to Sample 1 children in the same testing session: a language test, the (PPVT-III; Dunn & Dunn, 1997) and an early reading test, the Woodcock-Johnson Letter-Word Identification subscale (Woodcock et al., 2001). We expected a moderate to strong correlation between the Rasch-estimated child ability scores and these tests but expected that the correlations would be attenuated when compared to the correlation between the Rasch-estimated child ability scores and the Applied Problems subscale.

On the PPVT-III, Sample 1 children averaged a raw score of 58.31 (standard deviation of 21.84), while on the Letter-Word Identification subscale, Sample 1 children averaged a raw score of 12.49 (standard deviation of 7.16). The correlation between the Short Form and the PPVT-III was .64 ($p < .001$) and for Letter-Word Identification, also .64 ($p < .001$). In an ordinary least squares regression model with the Short Form score as the dependent variable and the other three tests scores as independent variables, Applied Problems was a stronger predictor of Short Form Rasch-estimated child ability scores ($\beta = .47$; $p < .001$) than either the PPVT ($\beta = .13$; $p < .001$) or the Letter-Word Identification ($\beta = .28$; $p < .001$). GLH testing confirmed that the relationship between Applied Problems and the Rasch-estimated child ability scores was statistically significantly different than the relationship between the PPVT and the Rasch-estimated child ability scores ($p < .001$) and between Letter-Word Identification and the Rasch-estimated child ability scores ($p < .001$). We interpret the higher correlation between the Rasch and Applied Problems scores and the GLH testing results as further evidence of good concurrent validity. Likewise, our findings regarding the relationship between the Rasch, PPVT and Letter-Word Identification scores provide evidence of good discriminant validity.

RQ 5: Do the Short Form items show DIF across subgroups of students?

The existence of DIF could mean that the Rasch-estimated ability scores from the Short Form are not representative of the individual's true mathematical competence (Zumbo, 2007). We examined the possibility of DIF separately within Samples 1 and 2. Conditioning on children's Rasch-estimated person ability score, we used logistic regression to test for DIF on each item among several subgroups – free/reduced lunch eligible vs. full-priced lunch students, home language (English, Spanish and Other), gender and grade level (preschool vs. kindergarten). We tested for DIF by language instead of by racial/ethnic group due to high correlations between some of the language and race/ethnicity variables and due to concerns raised in other literature that language can be a biasing factor on mathematics tests (Martiniello, 2008). In Sample 1, on each of the 19 items, we made three between-language item parameter comparisons (e.g. English vs. Spanish, Spanish vs. Other, and English vs. Other) and one comparison each for gender, free/reduced price lunch status and grade level, for a total 114 comparisons across all 19 items. In Sample 2, we made the same comparisons, except that there were no 'other language' speakers, some items were not given to prekindergarten students (due to the stop rule in the full sample), and we included a treatment group status indicator.[8] In total, we made 81 comparisons in Sample 2.

Given the number of statistical tests, to account for the emergence of false positives, we performed a Benjamini–Hochberg correction. This correction is a multiple comparison adjustment method shown to be superior to the traditional Bonferroni correction, particularly when many comparisons are made simultaneously (Cauffman & MacIntosh, 2006; Thissen, Steinberg, & Kuang, 2002; Williams, Jones, & Tukey, 1999). In the absence of DIF, we would expect the regression parameters on the demographic variables to be statistically indistinguishable from zero. In Sample 2, there were no statistically significant incidences of DIF. In Sample 1, across all 114 comparisons, we found seven statistically significant incidences of DIF. Three of the seven met the Bond and Fox (2007) threshold for substantively meaningful DIF, meaning an item for which the probability that one group would get it correct vs. a comparison group was more than .5 logits, conditioning on ability. Those items and comparisons were: item 1, English vs. Spanish ($\beta = .90$ logits; odds ratio = 2.45); item 2, English vs. Spanish ($\beta = 1.17$ logits; odds ratio = 3.23); item 6, prekindergarten vs. kindergarten ($\beta = -1.18$ logits; odds ratio = .31). Given that there were no items that showed DIF in both samples, we interpret these results as indicating that DIF in the Short Form is limited; that is, the Rasch-estimated ability scores from the Short Form are representative of the individual's true mathematical competence. Further, we find the limited detected DIF acceptable, given that the purpose of the assessment is to serve as an early mathematics screener and not to make any highly consequential, high-stakes decisions about student performance, subsequent classroom placements or programme admission.

RQ 6: Could a stop rule be used with the Short Form without substantial loss of information?

Stop rules are commonly used in early childhood assessments, including language, literacy and executive function tests (Dunn & Dunn, 1997; Frye, Zelazo, & Palfai, 1995; Gathercole & Pickering, 2000; Woodcock et al., 2001; Zelazo, 2006). They help to prevent test-taker fatigue and feelings of frustration, which is particularly beneficial in populations of young children, given their shorter attention spans. The basic principle behind their use is that, assuming items are arranged in order from least to most difficult, after a child misses a certain number of consecutive items, they are unlikely to get any subsequent harder items correct. Discontinuing the test thus results in little, if any, loss of information (Watson & Pelli, 1983).

The most current version of the full REMA employs a stop rule. More specifically, the full REMA is divided into geometry and numeracy sections and within the two sections, items are ordered from least to most difficult. After the child misses three consecutive items within a section that section is discontinued.

Because of the described advantages of stop rules, we explored whether applying the same test structure and stop rule on the REMA Short Form would result in loss of information that would substantially affect Rasch-estimated ability scores. Within Sample 1,[9] we divided the 19 items into geometry and numeracy sections, arranged the items within section by difficulty (least to most) and estimated the raw totals within section with the stop rule of three in place. The mean raw scores at the person-level for geometry and numeracy were extremely close with and without the stop rule (geometry – 6.36 with and 6.39 without; numeracy – 5.93 with and 6.11 without).

We also submitted the stop rule-adjusted data to a Rasch model. As shown in Table 5, descriptive statistics for Rasch-estimated person-ability scores were nearly

Table 5. Descriptive statistics, Rasch-estimated ability scores at the person-level with and without a stop rule of three consecutive wrong answers.

	N	Mean	Standard deviation	Min	Max
With stop rule	1930	−.17	1.41	−6.56	4.02
Without stop rule	1930	−.08	1.31	−6.38	3.89

identical between the two samples. Further, person-level estimated ability scores with and without the stop rule showed a correlation close to 1 ($r = .99$; $p < .001$), as did Rasch-estimated item difficulties ($r = .99$; $p < .001$).

These results suggest that dividing the REMA Short Form into geometry and numeracy sections, ordering items by Rasch-estimated difficulty and employing a stop rule of three is advantageous. Mimicking the format of the full REMA would be more consistent and less confusing for users of both tests. It also would help to prevent test fatigue among young children without any substantial or important loss of information about the child's ability.

Discussion

Early mathematics ability is a strong predictor of later academic outcomes, stronger than early reading ability and socio-emotional skills (Duncan et al., 2007). Yet, there are few validated, short measures mathematics skills that capture more than limited dimensions of this domain in the early childhood period. In this article, we evaluated the psychometric validity of a short form of a new, theory-based early mathematics assessment that taps a broader range of the mathematical competencies critical in early childhood. Our analyses indicated the Rasch model demonstrated adequate fit to the REMA Short Form items. The Short Form also provided information about students of differing ability levels, which is important given the intended purpose of the measure. Item functioning appeared robust across sampling differences. Calculated ability scores showed good concurrent validity with the full REMA and Woodcock-Johnson Applied Problems subscale and good discriminant validity with the Woodcock-Johnson Letter-Word Identification subscale and the PPVT. Items showed little to no evidence of DIF across subgroups (free/reduced lunch status, home language, gender and treatment group status). In sum, the REMA Short Form appears to be a psychometrically valid assessment of prekindergarten- and kindergarten-aged children's early mathematics skills.

Although this study was conducted in districts that were at least partially implementing a specific early mathematics curriculum (*Building Blocks*), neither the REMA nor REMA Short Form is tied to any specific curriculum. Both measures are consistent with *Common Core State Standards* that apply across curricula and are based on theory and empirical research regarding the developmental progressions, or trajectories, of mathematics competencies in the early years. Our DIF analysis provides quantitative support for this point; in our Short Form analysis, we found only one (out of 19) statistically significant instance of DIF by treatment or control status (where treatment children were all exposed to the *Building Blocks* curriculum) in Sample 1, and we found that DIF did not exist in Sample 2. In addition, also suggesting that the measures are not overly aligned with any specific curriculum, in two previous research studies, the full REMA has shown excellent concurrent validity with other early mathematics measures (Clements et al., 2011; Klein et al., 2000). In the present study, the Short Form likewise showed strong concurrent validity with the Applied Problems subscale. Further research on its use in settings that are implementing other curricula certainly would add to its evidence base. However, for theoretical reasons and based on empirical evidence, we contend that the REMA and REMA Short Form are valid for use in settings, independent of any specific curriculum. Nonetheless, a limitation of our study is that all Sample 1 and two-thirds of the Sample 2 settings were implementing the same curricula.

A notable strength of our study is that our results are derived from large, diverse samples of urban prekindergarten and kindergarten students. Many of the subgroups included in our study – particularly black, Hispanic, English Language Learners and free/reduced lunch children – are exactly those for whom achievement gaps in mathematics have been observed elsewhere and for whom observed gaps have raised concern (Clements et al., 2008; Ginsburg et al., 2008; Magnuson & Waldfogel, 2008; National Center for Education Statistics, 2000). The evidence presented in this paper suggests that the REMA Short Form is appropriate for use in diverse populations of young children. Future research is needed, however, to confirm that it works as well with rural and higher income populations, or children who are not in any formal preschool programme.

Given the increased interest in early mathematics education and assessment, our results are quite encouraging. Like its long-form parent, the REMA Short Form is one of the few measures of children's early mathematics skills that is theory based and includes geometry and spatial reasoning. It is relatively simple to administer, requiring a testing session of approximately 15–20 min. With a stop rule, administration time would be even shorter. Thus, on a practical level, within a single testing session, it can be combined with measures of child skills in other domains such as literacy and language.

Outside of the research domain, the REMA Short Form may be a useful tool for formative assessment by classroom teachers during the school year. The ease and brevity of administration, as well as its strong predictive relationship with the full REMA, suggest the Short Form could provide a useful snapshot of developmental mathematical competencies while also flagging students falling behind. Given classroom teachers are increasingly asked to conduct a broad range of assessments on children, the Short Form helps to reduce both teacher burden as well as child-testing fatigue. The Short Form could be used as a screening instrument for those entering school, or to ascertain which children in a class should be administered the full REMA for diagnostic purposes at any time. However, additional research on the utility of the Short Form in this context is needed. Further research is also needed on using the REMA Short Form with a stop rule.

Our study makes an important contribution to the literature, given the developmental importance of early mathematics skills in predicting later academic outcomes and given that gaps in mathematics achievement may begin in prekindergarten. Improving child outcomes and fully understanding the effects of early education interventions aimed at improving children's academic trajectories require measures that are both psychometrically sound and feasible, including in the important domain of mathematics skills. We believe the REMA Short Form is a step forward in this direction.

Notes

1. The Sample 1 school district was also the site of one of the school districts in Sample 2. Assessment data were collected in different years.
2. The overall study examined the efficacy of the Technology-enhanced, Research-based, Instruction, Assessment, and professional Development (TRIAD) model (for more detail on TRIAD, see Clements, 2007).
3. Subitising that involves the quick recognition of small sets is *perceptual subitising*. The REMA also assesses *conceptual subitising*, in which subsets are perceptually subitised and then combined, all quickly and automatically, such as when a '12 domino' is recog-

nised (Sarama & Clements, 2009). Both are fast, automatic and accurate quantification processes (not estimation nor involving explicit calculation).

4. As noted earlier, the Applied Problems subtest does not measure geometric and spatial capacities and researchers have raised some concerns regarding the test's appropriateness and sensitivity in use with young children. Nonetheless, it is very widely used and we include in the present study for concurrent validity purposes.
5. We recognise that using the Rasch model instead of a 2- or 3-parameter model assumes that items are equally discriminating and that the guessing parameter is zero (Weitzman, 1996). While we considered using a 2- or a 3-parameter Item Response Theory (IRT) model, we chose the Rasch model, given the centrality of Rasch modelling to the development of the full REMA. Further, from a practical perspective, Rasch modelling provides a simple, sample-independent conversion table from raw scores to ability scores that can be used by teachers and researchers who use the Short Form. By their nature, more complicated 2-parameter logistic model (PL) and 3-PL models do not have this practical advantage. Further, while a 2- or a 3-parameter IRT model might provide more accurate scoring (and this is debatable in the empirical literature; see Weitzman, 2008), the Short Form's intended purpose does not require such fine tuning.
6. An item with an infit or outfit statistic of >1.3 shows signs of underfit, meaning that it is not adequately distinguishing between children of differing abilities. Because no item showed inadequate infit and outfit statistics in both Samples 1 and 2, we allowed some flexibility in meeting this benchmark.
7. We report but do not interpret the standardised mean square infit and outfit statistics ('t standardized fit statistic (ZSTD)' in Table 3), as these tend to reject items when the sample size is large (Bond & Fox, 2007).
8. In Sample 1, treatment group status was equivalent to grade level, due to the multiple cohort design of that sample. This was not the case in Sample 2, where children within the same cohort were randomised to treatment/control group status.
9. We used only Sample 1 for this work because Sample 2 children took the full REMA using a stop rule. Therefore, we are unable to use Sample 2 to determine how much information was lost with the stop rule vs. without.

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