Learning Trajectories of Early Math

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Learning Trajectories

- Mathematics of children—representations and thinking of children as it developments naturally
- Activities matched to children’s development in each topic
- Therefore:
  - All within developmental capacities of children
  - Provide a natural “building block” to the next level
  - Provides mathematical building blocks for school success, because research based on typical higher-income children

Learning Trajectories: 3 Parts

1. Goal
2. Developmental Progression
3. Instructional Activities

Building Blocks Learning Trajectories LTs Keynote 6 for handout.key - May 12, 2014
Big Ideas

- Research: Teachers who study and focus on big ideas more successful
What Might Be Missed

- Learning Trajectories at the Core of the Common Core

Learning Trajectory for Counting

1st Goal: Accurate, confident object counting
Learning Trajectory for Counting

1st: Goal: Accurate, confident object counting

2nd: Developmental Progression…

• Precounter Says number words, not sequence: "one, two, four…".

• Chanter Says in sequence but may run together

• Reciter Verbal counting to 5, then 10

• Corresponder Counts correctly using 1-1 correspondence, at least up to 5 objects in a line.

• Counter (Small Numbers) Counts 1-5 objects in a line meaningfully (i.e., employ the cardinal rule)
Learning Trajectory for Counting

- Producer (Small Numbers) Counts out a collection up to 5
- Counter (10) Counts collections up to 10
- Counter and Producer (10+) and keeps track of unorganized collections

Learning Trajectory for Counting

- Counter from N
- Counter On Using Patterns
- Counter On Keeping Track
- Counter Forward and Back
Instructional Activities: 3rd Part of Learning Trajectories

Small Numbers and Counting

- Finger plays:
  - One, two, buckle…
  - When I was one…
    - When I was one, I was so small, (hold up 1 finger)
    - I could not speak a word at all. (shake head)
    - When I was two, I learned to talk. (hold up 2 fingers)
    - I learned to sing, I learned to walk. (point to mouth and feet)
    - When I was three, I grew and grew. (hold up 3 fingers)
    - Now I am four and so are you! (hold up 4 fingers)
- Later: Five Little Monkeys, etc.

Count and Move

Small Group Record Sheet
Building Blocks Math - PreK Assessment

<table>
<thead>
<tr>
<th>Trajectory Name</th>
<th>Description</th>
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<tbody>
<tr>
<td>Small Collection Naming</td>
<td>Can name collection of obj up to 2</td>
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<tr>
<td>Maker of Small Collections</td>
<td>Can make obj to 4</td>
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<tr>
<td>Counter (Small Numbers)</td>
<td>Can count obj up to 5 knows &quot;how many?&quot;</td>
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<tr>
<td>Perceptual Subitizer to 4</td>
<td>Can subitize up to 4</td>
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<tr>
<td>Perceptual Subitizer to 5</td>
<td>Can subitize up to 5</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Child's Name</th>
<th>Finds groups to:</th>
<th>Strategies/ Trajectory Level</th>
<th>Comments:</th>
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</table>
Activities for multiple goals

- What goals on number table?
  - Make and imagine small collections items *nonverbally*
  - Count by ones to 10
  - Know the last counting word tells “how many”
  - Count out (produce) a collection
  - Subitize (quickly “see” and label with a number)
  - Identify whether collections are the “same” number or which is “more” visually

Counting Games

- Count the dots and move that number of jumps
- *Connecting different representations of number!*

Road Race: Connecting Representations
Road Race Shape Counting - Another Variation

- Count the sides of a shape and move that number of jumps
- Connecting new concepts of number

Space Race Number Choice

- Choose the “better” of two numbers
- Comparing but also reasoning: Which is better in this case?

Arithmetic Sequence

- Encourage counting on from numeral
- Add numerals
- Addition “choice” game
- Two-digit addition

A Trajectory for Composing Geometric Shapes
Substitution Composer

- Finds different ways to fill a frame, emphasizing substitution relationships.

We Lay Groundwork Early…

- “First, I drew a triangle…
- then I had a trapezoid…
- then a parallelogram…
- And when I was having hexagons,
- I still had 10 triangles!”

Create A Scene
TRIAD II: Large-Scale Evaluation

TRIAD = Technology-enhanced, Research-based, Instruction, Assessment, and professional Development

Results: Child Assessment

- $F(1, 32) = 40.52$, $p = .000+$
- **T Scores:**
  - 50 Mean
  - 10 SD

Using the Learning Trajectories

It takes time. A teacher talks about interviewing a child for report cards:

“She was able to do verbal counting to 8, and then when she slowed down, she could get to 11. So I said, “Can you make me a group of 6?” And so she did. So then I added, I did 12, I think. She couldn’t do it.

Then I noted that, so now I’m thinking in the trajectories, I think she’s a “Counter (Small Numbers),” right? She’s on her way to being a “Counter (10).” She’s in between the two. So that’s what I was thinking of as I did this.”

—Pat, 2004

Web Sites (and article download)

TRIADscaleup.org
BuildingBlocksMath.org

“If we teach today as we taught yesterday, we rob our children of tomorrow.” –John Dewey

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