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Chapter 8

Mathematics for the whole child

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ALETHA: Math is just inappropriate for preschool. Children need to play, not memorize facts and drill on skills. It’s not like language, which you can help kids develop, well, more naturally or informally. Math you have to teach directly.

BRENDA: I agree, but that doesn’t make it inappropriate. Especially my preschoolers, who come from low-resource communities — they need that kind of direct instruction in math. Otherwise, they’ll be behind forever.

CATHY: It may be important, but do you need to teach it that way? I mean, aren’t they doing math when they build with blocks?

Which teacher do you agree with? Or do you have another opinion? What role is there for math early education? Should math be more teacher-directed or more child-centered? What are the best strategies to ensure all children are successful?

If you are similar to most people in the US, including most early childhood teachers, you have a firm (“gut”) feeling that math is hard, abstract, and at the opposite end of a spectrum from a vision of early childhood that features children at play, curious and creative. When mathematics is discussed in or out of school, it is likely you filter everything said through these feelings.

You are not alone. Although it is not true through its history (see Cohen, 1982), the US culture has become inhospitable for the development of positive attitudes and beliefs about mathematics. As just one example, all it takes to raise math anxiety in approximately 17 percent of the US population is … to show them “17%” (Ashcraft, 2006)!

Many adults believe “too much” mathematics is inappropriate for young children. Because they often struggled with mathematics themselves — frequently in badly taught, boring mathematics lessons — they believe that mathematics is difficult and inappropriately abstract. Further, their biases lead them to believe that mathematical experiences stand in opposition to play-based experiences.

There is some good news. These biases and the resultant fears have no basis in young children. Observe them learning mathematics in high-quality settings, or in their play, and you see that they love it.

Surprisingly, such fears and misunderstandings have been with our country for a long time. Let’s take a short historical trip.

Early math in the past

Frederick Froebel invented kindergarten. Kindergarten originally included a range of ages, so he invented present-day kindergarten and preschool. Froebel originally was a
crystallographer—he studied the shape and structure of crystals. Almost every aspect of his kindergarten crystallized into beautiful mathematical forms—the “universal, perfect, alternative language of geometric form” (Broserman, 1997). Its ultimate aim was to instill in children an understanding of the mathematically generated logic underlying creation. Froebel used “gifts” to teach children the geometric language of the universe. Cylinders, spheres, cubes, and other materials were arranged and moved to show geometric relationships. His “occupations” (activities children engaged in with the gifts) included explorations (e.g., spinning the solids in different orientations, showing how, for example, the spun cube can appear as a cylinder), puzzles, paper folding, and constructions. Children covered the faces of cubes with square tiles, and peeked them away to show parts, properties, and congruence. Shapes were arranged and rearranged on grids etched into each of the children’s tables, creating shifting, symmetric patterns or geometric borders.

Cubes that children had made into the chairs, ovens, and the like would be made into geometric design on these grids, and later laid into two rows of four each and expressed as “4 + 4.” In this way, connections were key: the “chair” became an aesthetic geometric design, which became a number sentence.

Unfortunately, not everyone observed the children engaged with and motivated to learn mathematics. Instead, with no real experience, based only on broad social theories, they drove mathematics out of the curriculum as “inappropriate” (Ballant, 1999).

Bureaucratic and commercial imperatives emerging from the institutionalization of early childhood education quashed most of the promising mathematical movements. For example, Edward Thorndike wished to emphasize health, so he replaced the first gift (small spheres) with a toothbrush and the first mathematical occupation with “sleep” (Broserman, 1997). And thus the mathematics was diluted and lost.

Consider another example. Many believe children should be playing, not doing mathematics. They should be building with blocks. Again, this reflects an unfortunate misunderstanding and loss of the historical root of those very blocks. The inventor of today’s unit blocks, Caroline Pratt (1948), designed them to teach mathematics (that’s why they have “units” and “double” and “half” units, and so forth). She tells of children making enough room for a horse to fit inside a stable. The teacher told Diana that she could have the horse when she had made a stable for it. She and Elizabeth began to build a small construction, but the horse did not fit. Diana had made a large stable with a low roof. After several unsuccessful attempts to get the horse in, she removed the roof, added blocks to tie walls to make the roof higher, and replaced the roof. She then tried to put into words what she had done. “Roof too small.” The teacher gave her new words, “high” and “low,” and she gave a new explanation to the other children.

Children create forms and structures that were planned, by Pratt and others, to embody mathematical relationships. For example, children may struggle with length relationships in finding a roof for a building. Length and equivalence are involved in substituting two shorter blocks for one long block. Children also consider height, area, and volume.

At least they may do so, implicitly and intuitively. Teachers such as Diana’s, who observe children and understand the mathematics, help children reflect on and further develop these intuitive ideas by discussing them, giving birth to explicit concepts and giving words to their actions. For example, children can be helped to distinguish between different quantities such as height, area, and volume. Three preschoolers made towers and
argued about whose was the biggest. Their teacher asked them if they meant whose was tallest (gesturing) or widest, or used the most blocks? The children were surprised to find that the tallest tower did not have the most blocks.

Thus, those who actually observe preschoolers engaging in high-quality mathematics see that they are naturally interested in and drawn to mathematics activities, situations, and problems. They love to engage in and play with mathematics. Those who quelled or vitiated mathematics in the curriculum were those who did not observe or interact with children (Balbuz, 1999). Just as we flower the preschool environment with books even before most children can read, just as we delight if a child wants to write her own name, we can easily ensure that mathematics is a joy throughout the preschool day.

Early math in the present: missing in action

Not much math learning happens in early childhood settings

Observations of the full day of 3-year-olds’ lives, across all settings, revealed remarkably few mathematics activities, lessons, or episodes of play with mathematical objects, with 60 percent of the children having no experience across 180 observations (Tudge and Doucet, 2004). Factors such as race-ethnicity, socio-economic status (SES), and setting (home or child care) did not significantly affect this low frequency. A study of four pre-K teachers from two settings revealed that little mathematics was presented, either directly or indirectly (Graham et al., 1997). Only one instance of informal mathematical activity with physical materials and few instances of informal or formal mathematics teaching was observed. Teachers stated that they believed that mathematics was important and that they engaged in mathematical discussions. It appears that selection of materials and activities such as puzzles, blocks, games, songs, and finger plays constituted mathematics instruction for these teachers.

How about the effects of programs that are ostensibly “complete” programs but fundamentally built upon literacy goals? In the OWL (Opening the World of Learning) curriculum, which includes mathematics in its all-day program, for every 360-minute day, only 58 seconds were devoted to math (Farzaneh et al., 2007). No children gained math skills, and those beginning with higher scores lost math skills over the year.

Little math is a big mistake

How important is it that math is often not present in today’s early childhood classrooms? We have shown that mathematical thinking comes naturally to children. Is not that enough? Not at all. Children do not develop that thinking unless it is intentionally supported.

Further, the mathematics achievement of American students compares unfavorably with the achievement of students from several other nations, even as early as first grade and kindergarten (Stigler et al., 1990). Some cross-national differences in informal mathematics knowledge appear as early as three to five years of age (Yuzawa et al., 1999). The knowledge gap is most pronounced in the performance of US children living in economically deprived urban communities (e.g. Griffin et al., 1994; Siegler, 1993).

These gaps are more important than previously realized. Early knowledge strongly affects later success in mathematics (Denton and West, 2002). Specific quantitative and
numerical knowledge in the years before first grade has been found to be a stronger predictor of later mathematics achievement than tests of intelligence or memory abilities (Krajewski, 2005). What children know early affects them for many years thereafter, into high school (National Mathematics Advisory Panel, 2008).

Thus, all children need a firm foundation in mathematics in the early years. This is especially true of children who live in poverty and who are members of linguistic and ethnic minority groups, because most have had significantly fewer opportunities to learn mathematics (Arnold and Doctoroff, 2003; Denton and West, 2002; Griffin et al., 1995; Jordan et al., 1992). Differences start early and widen (Alexander and Entwisle, 1988). If these children do not receive the best mathematics education possible, they are trapped in a trajectory of failure (Rouse et al., 2005) that closes the door to opportunities in all technical and most professional fields.

**Play “vs.” mathematics — the tragic — and false — dichotomy**

As described, educators of the past did not argue whether children should engage in play or mathematics — they created toys that embodied mathematics and taught mathematics. Further, children organically see the work through mathematical lenses. Researchers videotaped 90 4- to 5-year-old children, some from low-income families, others from middle-income families, as they played (Seo and Ginsburg, 2004). For example, a boy takes out all the beads in a box and puts them on a table. He states, “Look! I got one hundred!” He starts counting them to check that assertion. Others join in the counting and they do count up to one hundred, with minimal errors.

The range of mathematics is impressive: they classified, counted, and created patterns and shapes. Even more so is the frequency children engaged in math activities. About 88 percent of children engaged in at least one math activity during their play. Overall, the children showed at least one instance of mathematical activity during 43 percent of the minutes during which they were observed. Of course, these actions may have been just a brief episode within the observed minute of play, but there is little doubt that children are involved in mathematics during a considerable portion of their free play.

What does all this mean regarding children’s development? Such everyday foundational experiences form the intuitive, implicit conceptual foundation for later mathematics. Later, children represent and elaborate on these ideas — creating models of an everyday activity with mathematical objects, such as numbers and shapes; mathematical actions, such as counting or transforming shapes; and their structural relationships. We call this process “mathematization” (Sarama and Clements, 2009). That is, when children play a game and recognize that they cannot win on the next move, because they need a 7 and the largest number they can roll is a 6, they have represented the game situation with numbers and used mathematical reasoning. Children who recognize that a floor can be tiled with regular hexagons because “the angles fit together” have modeled an aspect of their world with geometry.

Further, recognizing the difference between foundational and mathematized experiences is necessary to avoid confusion about the type of activity in which children are engaged (Kronholz, 2000). Providing preschoolers with building blocks invites children to have foundational experiences with three-dimensional shapes. Asking them which blocks stack and which roll engages them in mathematizing those experiences, thinking about the properties of those blocks in geometric terms. Children need both types of experiences.
Unfortunately, adults often do not encourage children to reflect on the foundational experiences, using mathematical ideas and language, as we saw. Surprisingly, such math instruction actually increases the quality of young children’s play. Children in classrooms with stronger emphasis on literacy or math were more likely to be engaged at a higher quality level during free choice (play) time. Those in classrooms with an emphasis on both literacy and math were more likely to be engaged at a high quality level than those in classrooms with only one, or no, such emphasis (Aydogan et al., 2005). Thus, high-quality instruction in math and high-quality free play do not have to “compete” for time in the classroom. Doing both makes each richer, and children benefit in every way. Unfortunately, many adults believe that “open-ended free play” is good and “lessons” in math are not (Sarama, 2002; Sarama and DiBiase, 2004). They do not believe that preschoolers need specific math teaching (Clements and Sarama, 2009). They do not realize that they are depriving their children both of the joy and fascination of mathematics, and of higher-quality free play as well.

These and other examples bring us to the final fascinating and usually overlooked type of play: mathematical play. Here we do not mean play that involves mathematics—we’ve been talking about that throughout this article. We mean playing with mathematics itself.

Just after her fourth birthday, Abby is playing with three of the five identical toy train engines her father had brought home. Passing by, her mother asked, “Where are the other trains?” Although she left, Abby spoke to herself. “Oh, I have five. Ummm … [pointing to each engine] you are one, two, three. I’m missing ‘four’ and ‘five’—two are missing! [She played with the trains for another minute.] No, I have ‘one,’ ‘three,’ and ‘five.’ I’m missing ‘two’ and ‘four.’ I gotta find them two” (Clements and Sarama, 2009).

When Abby first figured out how many she was missing, she was using math in her play. But when she decided that she would renumber the three engines she had with her ‘one,’ ‘three,’ and ‘five’ and the missing engines ‘two’ and ‘four’ she was playing with the notion that the assignment of numbers to a collection of objects is arbitrary. She was also counting not just objects, but counting words themselves. She counted the words “four, five” to see if there were two missing, and then figured that counting the renumbered counting words “two” and “four” also yielded the result of “two.” She was playing with the idea that counting words themselves could be counted. Teachers well-versed in mathematics education will find and build on children’s boundless creativity in playing with mathematics itself.

Laissez-faire math is not enough

Although math can and should be playful and joyous, this does not mean that “letting children play” provides high-quality, or even barely adequate, mathematics education. Teachers who do not understand mathematics and how children learn mathematics are not able to recognize or enhance children’s spontaneous mathematical thinking.

This is unfortunate because mathematization is critical and is often neglected. Consider one more example. Teachers, even in middle school, approach the topics of parallelism and perpendicularity with trepidation. Then consider a preschool boy who is making the bottom floor of a block building. He lays two long blocks down, going in the same direction. Then he tries to bridge across the two ends with a short block. It does span across the long blocks, so he moves an end of one of the long block so it will reach. However, before he tries the short block again, he carefully adjusts the other end of the
long block. He seems to understand that parallel lines are the same distance apart at all points. He then confidently places the short block and quickly places many short blocks, creating the floor of his building (Seo and Ginsburg, 2004). Like this boy, many children intuitively use ideas that are theorems in action (Vergnaud, 1978). The boy even seemed to understand — in his actions — that parallel lines are always the same distance apart. Unfortunately, students often do not "understand" these concepts when they arrive in middle school. If teachers never help them to mathematize their theorems in action, to describe their understandings with the language of mathematics, the concepts will not become theorems in thought.

Further, even rich, everyday mathematics, properly discussed, are inadequate alone and must be complemented by intentional and sequenced instruction, especially for children who have not had many opportunities to learning math out of school. Traditional approaches to early childhood, such as "developmentally appropriate practice" (DAP) do not appear to increase children's mathematics learning (Van Horn et al., 2005). Programs based only on an "everyday" or "play" approach to mathematics education frequently show negligible gains. We need ways to keep the probable benefits of DAP, such as socio-emotional growth (Van Horn et al., 2005), and yet infuse the young child's day with interesting, equally appropriate, opportunities to engage in mathematical thinking (cf. Peisner-Feinberg et al., 2001). Teaching math indirectly through everyday activities does not predict achievement gains, whereas sequential, intentional group work does (Klein et al., 2008). Mathematization is requisite to basic mathematical ability. Adults must help children discuss and think about the mathematics they learn in their play. Further, mathematics is a hierarchical subject. Intentional teaching using a sequenced mathematics curriculum is an essential complement to rich, scaffolded free play. This is especially important for children from low-resource communities.

High-quality, explicit, and sequential teaching should be the core of children's mathematical experiences. This helps children learn, helps teachers see the mathematical potential of other, everyday activities and, as we have seen, even promotes higher-quality play.

Everyone wins.

Learning trajectories: the mathematics of children

The backbone of high-quality teaching and learning is the use of research-based learning trajectories. Children generally follow natural developmental trajectories in learning mathematics. When teachers understand these trajectories, and sequence activities based on them, they can build powerful mathematics learning environments.

Each learning trajectory has three parts: a goal, a developmental progression, and instructional activities. To attain a certain mathematical competence in a given topic or domain (the goal), children learn each successive level (the developmental progression), aided by tasks (instructional activities) designed to build the mental actions-on-objects that enable thinking at each higher level (Clements and Sarama, 2009; Sarama and Clements, 2009).

For example, research has revealed a developmental progression of children's ability to compose geometric shapes, both in 2-dimensional puzzle play and free play (Clements and Sarama, 2009; Sarama and Clements, 2009). Children at first are unable to combine shapes and can solve only the simplest puzzles, in which individual shapes only touch at
Figure 8.1 Examples of shape composition activities from the Building Blocks curriculum (Clements and Sarama, 2007a)

their corners (Figure 8.1c). They gradually learn to see both individual pieces and a “whole” and learn that parts can make a whole and still remain parts (Figure 8.1b). Most preschoolers can solve puzzles by trial and error, and make pictures with shapes placed next to one another. With experience, they gradually learn to combine shapes to make larger shapes. They become increasing intentional, building mental images of the shapes and their attributes, such as side length and angles (Figure 8.1c). They should do this with
physical blocks and computer shapes. (Only the latter are pictured in Figure 8.1, but both are important.) Computer versions do have advantages, including giving immediate feedback, such as highlighting shapes that do not fit and making those shapes transparent so children can see the outline puzzle “underneath” them. Further, children often talk more and explain more of what they are doing on computers than when using other materials (Clements and Sarama, 2009).

Teachers who understand this learning trajectory can provide puzzles and other tasks and environments that are truly developmentally appropriate—challenging but achievable. They can see the advance of children’s thinking in free play in 2D and 3D (block building) contexts and promote it with challenges (e.g. “You ran out of those rectangular blocks. Can you make more by combining some of these?”).

Curricula based on such learning trajectories, such as Building Blocks (Clements and Sarama, 2007a), have shown consistent, large gains compared to other curricula and approaches (Clements and Sarama, 2007b; 2008). This is true even when scaled up to entire school districts (Clements et al., 2011; Sarama et al., 2008).

Final words

Is it true that the experiences of R. Buckminster Fuller, Frank Lloyd Wright, and Paul Kale in Froebelian kindergartens are the foundation of all their creative work, as Brosterman (1997) claimed? Whether or not this is true, it is clear that mathematics in the early years is not a recent invention. But over centuries those without knowledge of children have eliminated these opportunities for young children. Today, many adults have biases against mathematics, and these adults militate against mathematics for their children.

But those who observe children can see they love mathematics. Given the opportunity, children organically see the world through mathematical eyes. They explore, discover, and discuss mathematics. They also show mathematics in their play and play with mathematics. They do so more if their teachers use a comprehensive, research-based mathematics curriculum. Just as important, children whose teachers use intentional teaching and sequenced mathematics curricula engage in more high-level social dramatic play than children whose teachers do not.

Children in preschool can learn more mathematics, in multiple ways that honor their unique ways of thinking, than was previously thought possible. Learning trajectories can support children’s learning, as well as assessment and curriculum development and enactment. Children whose teachers use research-based learning trajectories demonstrate higher levels of mathematical reasoning. They help children learn the language of mathematics. Current research in learning trajectories points the way toward more effective and efficient, yet creative and enjoyable mathematics learning through culturally relevant and developmentally appropriate curricula and assessment.

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