Discussion from a Mathematics Education Perspective

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Abstract

In a review of the special issue, we conclude that the articles are research gems in the domain of preschool mathematics education. Most share several features, such as their perspective on research methodology and their view of mathematics thinking and learning. They address the cognitive architecture and processes, and the developmental levels through which young children pass (the core constituent of research-based learning trajectories). Most of the authors use similar constructs and even measures, a sign of a maturing research field. We hope such confluence begins to extend across groups from different disciplines. This could also serve the purpose of widening the research methodologies with those from mathematic education, such as those including theoretically-designed interventions and experimental designs.

Keywords

Mathematics education, early math, teaching and learning, learning trajectories, mathematical cognition.

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This special issue includes research gems in the domain of preschool mathematics education. Most share several features, such as their perspective on research methodology and their view of mathematics thinking and learning. They address the cognitive architecture and processes, and the developmental levels through which young children pass.

Magnitude Representations and Counting Skill

Sophie Batchelor, Sarah Keeble, and Camilla Gilmore make a substantive contribution to the field in their study of the relationship between counting concepts and skills on the one hand and magnitude comparison on the other. Both these are foundational to early mathematics learning, and we know much about how each develops (e.g., "developmental progressions" of learning trajectories, Clements & Sarama, 2014; Sarama & Clements, 2009), but we know less about the relationships between such developmental progressions. Such relationships are just as important as the separate progressions to our understanding of the psychology and education of young learners of mathematics. Thus, this study not only provides information about this particular relationship but also stands as an example of what future researchers might investigate for other similar relationships.

Creating a novel task and administering this task to an appropriate range of children (i.e., including younger children), they found that children could map between nonsymbolic quantities and those (smaller) number words for which they had built cardinal meaning, even if they did not yet exhibit an understanding of the general cardinality principle of counting. These findings are consistent with learning trajectories that posit that children learning counting skills such as a count list, correspondence, and cardinality in a predictable but interleaved fashion, with development in each supporting the development of the others. Also, in these trajectories, cardinality emerges for small numbers before the general cardinality principle is constructed (Sarama & Clements, 2009). We also find subitizing to contribute to the connections between counting actions and cardinality, leading us to ask, were there assessments of recognition-of-number or subitizing? Is that evidence showing that these children, even the older ones, could not subitize 4 or 5?

This raises another issue regarding verbal counting, cardinality, and meanings of number. Although our own developmental progressions (the core of learning trajectories) are consistent with the importance of cardinality (Clements & Sarama, 2014; Sarama & Clements, 2009), we believe it may be a slight overstatement to say that without this construct, children are not about to "understand the meanings of these words" (and the authors of the article may agree—recall their review of research suggesting "that even very inexperienced counters have some knowledge of large number word mappings"). Again, we agree regarding the cardinal meaning. But they may very well have an ordinal meaning and a sufficiently strong ordinal sense may imbue the words with an intuition about "how manyness" that a strict test of cardinality would not indicate. This does not weaken the findings here, but simply suggests that future research should more closely examine the understanding of children who, for example, not only can count verbally ('count list knowledge'—see Torbeyns, Gilmore, and Verschaffel's introduction on this issue), but also maintain one-to-one correspondence (as the authors say, "the complex, multi-componential nature of counting skill).

Although Batchelor, Keeble, and Gilmore make a good argument that *some* "How Many?" tasks are weak indicators of cardinality, we believe that the production ('Give-a-Number') task represents yet a higher level of thinking (Sarama & Clements, 2009), in that it requires a new executive control procedure that monitors the move-and-count procedure from the previous level to check, at the production of each count word, if that word matches the goal number; if so, it stops the move-and-count procedure. This requires the (at least implicit) understanding that a cardinal word sets such a goal. Again, this in no way invalidates the authors' assumption that such tasks indicate cardinality, but only raises a question regarding the possibility that this higher level of thinking may be involved.

As a final side note, in our early piloting, we did not find the 'guessing games' engendered learning, but that board games, other reasoning-oriented counting activities, and subitizing tasks did. This may be important in that some discussions use the term "numerical estimation" to mean the same research-based skill, and not other types of estimation. (Other types include estimation of the number of pictures of objects in a random display, which our early informal work—supported by Batchelor, Keeble and Gilmore's rigorous study—suggested was not pedagogically useful *until* children achieved adequate counting and cardinality concepts and skills). We look forward to more research to evaluate these and other possibilities.

Given the role that numerals may play in children's development and abstraction of number concepts, we hope that Batchelor, Keeble and Gilmore will see if the results are similar when numerals are added to the "symbolic" card (which now has just a picture of a box, with the only symbol a verbal number word) in their future research endeavors. Finally, the finding that achieving the level of thinking that includes cardinality (Clements & Sarama, 2014) has significant ramifications for nonsymbolic-symbolic mappings provides both researchers and practitioners a valuable guide to preschoolers' thinking and learning.

Familiarity with Numbers and Number Line Estimation

Ebersbach, Luwel and Verschaffel also use the term "numerical estimation" but are clear that in their work this applies to number line estimation. Children's performing and learning of a certain type of number-line estimation has received substantial attention, as it appears correlated with (e.g., Siegler & Booth, 2004; Zorzi, Priftis, & Umiltà, 2002), predictive of (e.g., Praet & Desoete, 2014), and causally related to (Booth & Siegler, 2008; Laski & Yu, 2014) later achievement in mathematics.

Ebersbach, Luwel and Verschaffel provide a well-developed discussion of previous research on number line estimation and the issues that have arisen. Based on this, they designed research that carefully measured familiarity of numbers beyond simple verbal counting from one and number line estimation in bounded and unbounded conditions. They also computed a comprehensive set of analyses, yielding a cogent case that older children who were familiar with a larger number range estimated more accurately and with less variability across the full range of numbers. Surprisingly, labeling the upper reference point (i.e., “100”) had little effect. Instead, children here appeared to establish an internal reference and iterated it up to the target point. This was termed a “dead reckoning strategy.” (We know “dead reckoning” mainly from a triangulation of a position, but understand the use here. Note that the phrase was a misreading of “ded. reckoning” — abbreviation for “deductive reckoning”— with “reckoning” based on the Dutch root meaning “to count up”. Therefore, mathematical logic and content were always the basis of this phrase.)

Such a notion connects to two developmental progressions of learning trajectories we developed previously. In cooperation with Gail Brade (2003), we created a developmental progression for estimation of (discrete) numerosities (vs. measurement/number line or computational estimation, cf. Sowder, 1992), a central theme of which was the construction of quantitative categories and units (exact, such as subitized numbers, or approximate, as estimated units-of-units) and then iteration of those units-of-units. For both Ebersbach, Luwel and

Verschaffel's number line estimation and the estimation of discrete numerosities, then, a nascent conceptual scheme for decades qua units-of-units may underlie the "familiarity" with numbers to 100 and be one path toward the building of the "mental number line/path" that research identifies as a critical component of early numerical competence (Booth & Siegler, 2008; Clements & Sarama, 2014; Laski & Yu, 2014; Praet & Desoete, 2014; Sarama & Clements, 2009; Siegler & Booth, 2004; Zorzi et al., 2002). However, it is not clear this is the only, or best, path. Given there is research that suggests that the majority of first graders' number line estimations use end points and midpoints, consistent with a proportional reasoning strategy (Rouder & Geary, 2014), it seems that future research needs to examine groups as well as individuals and carefully assess the role of a variety of contextual (task) factors (including Ebersbach, Luwel and Verschaffel's' useful suggestions for changes of conditions). We need experiments to establish true causal connections between "familiarity with numbers," "number line estimation" and other tasks, and thus to establish firmer guidelines for curriculum development and teaching. We hope the authors and others continuing this research use a variety of qualitative and quantitative approaches to investigate the cognitive processes children are using and, eventually, the learning trajectories (including both developmental progressions and correlated instructional tasks and strategies) for these competencies.

Predicting Mathematics Achievement with Early Spontaneous Focusing on Numerosity, Subitizing, and Counting Skills

A recent flurry of research has provided considerable information about what early skills predict later mathematics achievement (e.g., Bull, Espy, Wiebe, Sheffield, & Nelson, 2011; Claessens & Dowsett, 2014; Duncan & Magnuson, 2011; Geary, 2011; Hindman, 2013; Martin, Cirino, Sharp, & Barnes, 2014; Romano, Babchishin, Pagani, & Kohen, 2010; Sabol & Pianta, 2012; Sasser, Bierman, & Heinrichs, 2014; Toll & Van Luit, 2014; Van der Ven, Kroesbergen, Boom, & Leseman, 2012; Watts, Duncan, Siegler, & Davis-Kean, 2014, and, of course, several studies in this special issue). In this vein, Hannula- Sormunen, Lehtinen, and Räsänen provide an extensive review of previous work (mostly in psychology, less in mathematics education) and build upon this base, and especially their own previous work regarding children's spontaneous focus on numerosity (SFON).

Hannula- Sormunen, Lehtinen, and Räsänen's comprehensive analyses provide a clear picture of their findings, that SFON and verbal number sequence skills separately predict mathematical skills six years later, whereas subitizing measured one year earlier followed an indirect path to later skills, through both SFON and verbal counting skills. Controlling for non-verbal IQ, SFON but not verbal counting remained significantly predictive. We note that the phrase "subitizing-based enumeration" seems somewhat misleading, given that enumeration usually applies to one-by-one counting or listing, whereas subitizing refers to identification of a group without such enumeration.

Given that the measure of nonverbal IQ appears weakly related to relevant variables, future research might use other measures of general cognitive skills. Also, verbal counting is but one component of counting skills, and future research might employ assessments that measure other, arguably more mathematically central, aspects of object counting and counting strategies (e.g., Clements & Sarama, 2014; Sarama & Clements, 2009). The authors may be correct that SFON is more predictive than counting, but before a conclusion such as "a new perspective to the relationship between counting and the development of mathematical skills" is accepted, additional measures should be employed and a greater number of children should be studied.

Assessing Quantitative Reasoning

Nunes, Bryant, and Evans present a test of quantitative reasoning that was predictive of later mathematics learning. Theoretically- and empirically-supported tasks represented four aspects of such reasoning — thinking about relations between quantities. Such quantitative and mathematical reasoning are distinct from other abilities, such as counting, subitizing or SFON, or perceptually-based comparisons that are frequently used in predictive studies.

The authors' previous work shows that mathematical reasoning is a significant predictor of much later mathematical achievement. This study builds on those results by showing that early quantitative reasoning predicts that mathematical reasoning. Indeed, both children's quantitative reasoning and their arithmetical skills were strong and independent predictors of their mathematical achievement over subsequent years. Further evidence that these results have direct practical implications for education is provided by the authors' successful training studies (e.g., Nunes et al., 2007). Together, these are interesting and important findings.

Nunes, Bryant, and Evans are clear about their reasons for controlling for more general measures of cognitive functioning, such as general cognitive ability and working memory. This is a valid perspective, but we may wish to ponder if this frequent practice in the research literature assumes that those cognitive processes are fixed and quantitative reasoning processes are not. An alternative conceptualization would be to examine the interrelationships among and malleability of all those processes and knowledge components. If the assessments are strictly valid, then removing variance accounted for by general cognitive ability and working memory may make sense. But what if quite a bit of mathematical (even quantitative) competencies are measured by these assessments? This could change the theoretical and practice implications of many of these analyses.

Early Numeracy and Literacy

The domains of mathematics and literacy have repeatedly shown to be connected (e.g., Vukovic, 2012), with suggestions (e.g., longitudinal research) (Clements & Sarama, 2014; Sarama & Clements, 2009) of causation and even some documenting causal relationships (Sarama, Lange, Clements, & Wolfe, 2012). As an example of longitudinal correlational research, preschoolers' narrative abilities, particularly their ability to convey all the main events of the story, offer a perspective on the events in the story, and relate the main events of the story through use of conjunctions, predict mathematics achievement two years later (O'Neill, Pearce, & Pick, 2004). In the other longitudinal direction is the widely-cited study across six studies showing that early knowledge of mathematics predicted not only success in math, but also later success in reading; the strongest predictors of later achievement were school-entry math and attention skills (Duncan et al., 2007), a result that has been replicated (Romano et al., 2010).

Purpura and Napoli contribute to this field by identifying particular domains of mathematics and language that appear to be the core components of such connections and investigating their interrelationships, including mediational relationships. Within their three-phase model, they argue that language is related to numeral knowledge strictly as it affects informal numeracy, and similarly that print knowledge mediates the relationship between informal numeracy and knowledge of numerals. Further, we are not sure if the "phase" model actually involves phases if two of the three develop in parallel. The authors later draw an analogy between these phases and three domains of literacy, and perhaps that term is more apropos.

The first of these hypotheses was well supported, with the relation of language on numeral knowledge fully mediated by informal numeracy. This is a cogent argument, then, with language playing an early role connecting number words to various nascent concepts of quantity. Of course, these findings are specific and do not indicate a limited role that language plays in children's development of mathematical thinking. Although we understand the authors' argument concerning topics other than numeracy, there is a real concern that continuing studies such as this propagates with the domain of research but especially within the domain of practice, that numeracy equates with, or is the only truly important component of, mathematical thinking for young children.

The second hypothesis also received support, with a significant mediation effect of print knowledge on the relation between informal numeracy and numeral knowledge, albeit this was complementary or partial mediation. Thus, print knowledge such as knowledge of letters (cf. Austin, Blevins-Knabe, Ota, Rowe, & Lindauer, 2010) may provide a cognitive framework related to the role of codes for connecting early intuitive and verbal number competences to the symbolic system of numerals. (The authors' footnote 1 states that 'the societal determination that "3" represents ••• could have very well be "2" to represent •••: The specific formulation of the Arabic numerals does not hold a direct meaning': Although this probably true for most, it is interesting that, historically, Brahmi numerals were 1, 2, and 3 horizontal lines, etc., which developed into those Arabic numerals. Some present-day instructional techniques use those geometric structures.)

Given the mixed results regarding the predictive power of literacy skills to later mathematics learning (e.g., Passolunghi, Vercelloni, & Schadee, 2007), Purpura and Napoli offer welcome specificity.

Models such as these are complex, and some details may have been omitted for the sake of space. We need to know if Purpura and Napoli used only statistical significance at step one and whether the presence of multiple mediators in a single model may have changed the nature of the relationships found when these were entered alone. Such SEM analyses can produce different, but just as statistically satisfying models with different relationships. As a final note, we would prefer that the term “usually” should precede the statements about when children learn these skills and concepts.

Parent Literacy and Numeracy Expectations

As did Purpura and Napoli, Segers, Kleemans and Verhoeven examine the early effects of both numeracy and literacy environments, albeit within the home environment. They investigate the issue of whether the degree of numeracy in the home is predictive of children’s numeracy skills and whether it remains predictive of later learning of mathematics when one accounts for the corresponding degree of literacy in the home. Importantly, the numeracy environment predicted early numeracy while controlling for cognitive and linguistic child factors, even including the home literacy environment. Somewhat surprisingly, the home literacy environment was not predictive when child factors were taken into account. These results thus evidence the unique contribution of the home numeracy environment to young children’s early numeracy development.

Segers, Kleemans and Verhoeven report comprehensive psychometrics regarding the home survey, but we still worry a bit about the veracity of the self-report data and hope future studies can include direct observation. Although their discussion of missing data was equally comprehensive, the low response rate (50% of those contacted) remains a caveat regarding generalizability.

Although the relationships among literacy and mathematical concepts and skills are important, mathematics has its own semantics and syntax. The authors say that the “order of words in a sentence convey its meaning (“The girls walks the dog.” vs. “The dog walks the girl.”), as does the order of the numbers and operations in a calculation (“3 + 4/2” vs. “2 + 3/4”).”

However, order is not always unambiguous in mathematical notion, and different rule systems apply in “school” versus professional mathematics.

Finally, we were a bit surprised that a measure of phonological awareness included letter naming, often considered a separate construct (Pearson & Hiebert, 2010). Separating these and other factors with a larger population may be useful. Future research also might take bilingualism and other cultural factors into account, as the authors note. Given the bidirectional influences in many studies in this issue’s and other publications, such co-mutual relationships among various numeracy and literacy competencies and home resources might also be investigated.

Segers, Kleemans and Verhoeven’s findings add support and urgency to the appeals for families to become more aware of the need for early mathematics stimulation and, of course, to act on this awareness (e.g., Bodovsky & Youn, 2012; Clements & Sarama, 2014; National Research Council, 2009; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009), although Segers, Kleemans and Verhoeven’s cautions regarding the (concurrent) correlational nature of their research are well taken.

Final Worlds

It is heartening to see many of the authors using similar constructs and even measures, as these are signs of a maturing research field. At the same time, we note that such confluence does not extend across groups from different disciplines. Fully shared constructs and measures are necessary for scientific progress. Mathematics education is an interdisciplinary field and communicating across those disciplines will be increasingly important to continued progress.

The studies presented in this special issue make serious contributions to that field, especially as they focus on the early years of preschool. However, from the perspective of mathematics education, they stop short of considering the context. Would the concurrent relationships hold in the contexts of cultures (both national and local, including individual classrooms)? Would distinct curricula alter especially the longitudinal results? Beyond curriculum as standards and sequenced activities (e.g., educational texts), we speak here of curriculum defined as the complete learning experience of a school, including content, instructional methods, materials, resources, norms and values, expectations, and even school organization and culture. The ninth tenet of our theory of hierarchic interactionalism, environment and culture, posits that "environment and culture affect the pace and direction of the developmental courses…. Because environment, culture, and education affect developmental progressions, there is no single or "ideal" developmental progression, and thus learning trajectory, for a topic. Universal developmental factors interact with culture and mathematical content, so the number of paths is not unlimited, but, for example, educational innovations may establish new, potentially more advantageous, sequences" (Sarama & Clements, 2009, pp. 23-24).

Thus, we agree with several authors who mentioned the need for theoretically-designed interventions and experimental designs. We also argue this is not only necessary to establish causation, but also because the interrelationships among cognitive components may be altered by different educational contexts. As Richard Feynman ("What I cannot create, I do not understand"; "If it disagrees with experiment it is wrong", http://en.wikiquote.org/wiki/Richard_Feynman) and others consistently remind us, science is best conducted by probing the phenomena in question. Finally, extant work on interventions is missing from most authors' review of the literature. Such isolation of fields of research weakens all of them. Mathematics education should not be an "implication" tagged on to the end of studies from developmental and cognitive psychology. Mathematics education research and cognitive research should be interwoven enterprises. For example, mathematics education research emphasizing curriculum research (Clements, 2007) is valuable because it is result-centered, rather than theory-centered, and thus minimizes seductive theory-confirming strategies that tend to insidiously replace the intended theory-testing strategies, and maximizes strategies that attempt to produce specified patterns of data and thus mitigate confirmation bias, stimulating creative development of theory (Greenwald, Pratkanis, Leippe, & Baumgardner, 1986). Nevertheless, this issue's studies contribute to the growing body of knowledge that is strongly suggestive of those early competencies on which to build future mathematics learning.

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