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CHAPTER 12

WALKING THE SAME *BROAD* PATH (WITH SIDE TRIPS)

Response to Comments

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Researchers must become "thick-skinned" in reading criticisms of their writing. Reviewers can be harsh, even brutal, but most writers admit that they have learned more from such criticisms than from almost anything else they have read.

It is with particular pleasure, then, to read these illuminating—*and* kind—reviews by premier researchers in the field. We think this book speaks to the growing maturity of the field, in that there is sufficient empirical evidence that guides most of us to walk the same path, albeit with considerable room for differences in perspectives and foci (e.g., what bumps are most troubling) and allowances for important side trips.

Art Baroody, David Purpura, and Erin Reid emphasize the importance of the "big ideas" of mathematics—a notion with which we agree. Indeed, we

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have previously co-authored a structure for these big ideas in collaboration with Baroody in the past (see especially Part 1 in Clements, Sarama, & DiBiase, 2004) and have incorporated such structures directly and specifically into our notion of learning trajectories (Clements & Sarama, 2009; Sarama & Clements, in press). Baroody, Purpura, and Reid correctly go beyond describing these big ideas to the additional implication that standards, curricula, professional development, and teaching must not only be structured around such big ideas but should help *students* recognize and use these ideas to connect seemingly disparate domains of mathematics.

We believe that there are few practical differences between our position and that of Baroody, Purpura, and Reid on the educational implications. The main difference seems to rest with the nature of dichotomies. We agree that "genuine philosophical differences about teaching and learning" exist and are of fundamental importance. Still, we do not believe that the "meaningful teaching/memorization versus teaching/memorizing by rote" *dichotomy* is completely valid and useful. That is, as a continuum, this raises important issues. Research substantiates that too many U.S. mathematics classrooms are far closer to the "rote" end, with too little balance. Here we appear to agree. However, we regard a dichotomy as two disparate entities that are *opposed, entirely different,* and *mutually exclusive.* Only here do we disagree with Baroody, Purpura, and Reid. Although "rote" would rarely be in our description (except for certain limited cases of social-arbitrary knowledge such as number words to 10), we do not interpret meaningful learning as *opposed* to or mutually exclusive of learning through "repeated experiencing" or what Vygotsky (1934/1986) called spontaneous (everyday) knowledge.

Is this simply semantics? Somewhat. However, when we see how many teachers reject *any* repetition (practice, or what we prefer to call "repeated experiencing") *because* one has to "choose a side"—either "child centered" or "drill-and-kill,"—we believe there are significant practical ramifications to what we still call a false dichotomy. Of course, there are those who believe all early learning is by "rote," and only later becomes meaningful—an even more harmful application of the false dichotomy.

In this, we seem to be in agreement with Baroody, Purpura, and Reid, at least when they discuss other issues, such as "student-centered or teacher-directed instruction." However, these authors say we "recommend" a direct instruction approach. This is an overstatement. We cite research supporting that approach for children *with learning disabilities* or difficulties, and then we provide an extended discussion of reinterpretations (it is *explicit,* rather than "direct" instruction is supported by research), modifications, and alternate approaches. We believe that most of our recommendations are consistent with what the authors call the "investigative approach" (e.g., we state that "research supports the notion that inventing one's own procedures is often a good first phase in ensuring these advantages").

To repeat, we believe our practical implications are often consistent with those of Baroody, Purpura, and Reid. Further, we appreciate the additional details that their discussion offered, which we were not able to provide due to restrictions on the length of our chapter.

To resolve what may be seen as a difference between our position and that of Baroody, Purpura, and Reid in the notion of "practice," the reader should recognize that they have continued to define practice as drill-and-practice, whereas we redefined it as repeated experiencing, which *includes* all of what they call "meaningful learning/memorization." Repeated experiencing is not a euphemism for meaningless drill, but rather refers to experiencing the idea in multiple varied contexts, using it to solve problems (what Wirtz called "practice at the problem-solving level" [1974]), and incorporating it into the structure of mathematics. The research we cite on practice is not limited to meaningless drill, but the attainment of expertise. We do not accept our recommendations as a "straw man" of weak, unsubstantial, drill.

The authors appear to misunderstand Hiebert and Grouw's (2007) chapter and our brief review of it—nothing in either suggests that "teaching for skills only" is recommeded—it is simply a common practice that they describe. Indeed, if there were anything close to a true dichotomy, that of Hiebert and Grouws would be it.

We sincerely thank Anita Wager and Tom Carpenter for emphasizing a sociocultural perspective. Because we agree wholeheartedly with what they say, we feel we must point out that this perspective is honored in other publications (Clements & Sarama, 2009; Sarama & Clements, 2009), albeit not developed as thoroughly and as well as by others, including Wager and Carpenter.

Thus, we agree that cultural context matters to learning trajectories. We admit that Wager and Carpenter's statement that our learning trajectories are a cognitive construct and believe their caveats are well placed. However, we wish to add that our broader theory, *hierarchic interactionalism*, although also cognitivist, also considers sociocultural factors. We believe that thinking such factors have a profound effect and that "Different developmental courses are possible, depending on individual, environmental, and social confluences (Clements, Battista, & Sarama, 2001; Confrey & Kazak, 2006)" (Sarama & Clements, 2009, pp. 22–23). Further, one of the dozen tenets of the theory is precisely that "Environment and culture affect the pace and direction of the developmental courses" and "because environment, culture, and education affect developmental progressions, there is no single or 'ideal' developmental progression, and thus learning trajectory" (p. 23). To repeat, we agree that our chapter did not develop this perspective, and thus contend that Wager and Carpenter's comments are a necessary and useful complement to our review.

Wager and Carpenter correctly question the "age" column in our presentation structure for learning trajectories. In works focused on learning trajectories, we provide an elaboration, which is relevant to their concern. For example, we have previously stated that "the ages in the table are typical ages [at which] children develop these ideas. *But these are rough guides only*—children differ widely. Furthermore, the ages below are lower bounds on what children achieve without instruction. So, these *are "starting levels," not goals.* We have found that children who are provided high-quality mathematics experiences are capable of developing to levels one or more years beyond their peers" (Clements & Sarama, 2009, p. 6, emphasis in original).

More important, we would not agree with Wager and Carpenter's characterization of age as a "component"—it is simply a heuristic. That is, age is necessary in the initial formation of learning trajectories. Within and across topics, most studies involved children of a single age. Thus, we use age as a way of sequencing the levels in nascent learning trajectories. This is an explanation and also a caveat: Not all trajectories have been tested as comprehensive entities (ideally, with both cross-sectional and longitudinal designs). Thus, much work needs to be done. This work should be sensitive to Wager and Carpenter's concerns regarding the situated nature of children's learning and the need for a "critical view." However, we also believe that rejecting the view of children as "deviant" ("blaming the victim") does but should not imply that we, as an educational community, should accept that there are substantially different learning trajectories for some groups of children. First, this is too often translated into "my children learn more slowly and differently"—an insidious trap of low expectations disguised as sensitivity to cultural and individual differences. Second, we believe that the admittedly cognitive core of learning trajectories is valid with different cultural contexts.

Thus, we believe we are consistent with Wager and Carpenter in stating that learning trajectories can be the cognitive core that is instantiated in different ways in different sociocultural settings. The minor disagreements should not detract us from our appreciation of their perspective in expanding this core in necessary ways by emphasizing the importance of the sociocultural contexts of education and the growing research base that guides educators in using it to promote learning and equity. As such, their chapter is a critical complement to our own.

Karen Fuson's comments also go beyond our chapter to add valuable information about ways that the Common Core State Standards (CCSS) can improve teaching and learning in early and elementary schools. Fuson was a key figure in forming the algebraic thinking and number base-ten developmental progressions (what she calls "learning paths") on which those topics in the CCSS were based, and her discussion of them adds useful details to our review.

Because Fuson's chapter is an extension, there are few issues to which we need to respond. For example, we are pleased that her research is consistent with our theory of hierarchic interactionalism and its learning trajectories. One minor note: We believe the phrase "learning trajectories" is more accurate and useful than learning paths because the construct of learning trajectories (a) has an established theoretical and empirical history (Simon, 1995) with considerable research and development work (Clements & Sarama, 2004); (b) implies a direction and progression (whereas "paths" can simply meander); and (c) includes three well-defined components of goal, developmental progression, and instructional tasks.

Another issue needing clarification is that of geometric shapes. Although we agree with Fuson on the importance of many different categories of shapes, we disagree with her notion of what are "central 2-D shapes" and believe she has misinterpreted our chapter. First, the learning trajectory pictured in our chapter involves only shape composition. Second, the shapes for that learning trajectory are simply illustrations using the most commonly available shapes; the complete learning trajectory uses shapes with only multiples of 45° even in preschool (see Clements & Sarama, 2007, 2009). Third, our learning trajectories for geometric shapes (beyond simply composition) include all the shape categories she mentions as being illustrated in our chapter, all the shape categories she describes as "central," and many others. Children are miseducated when they are overexposed to any limited set of shapes, no matter how useful for some purposes (Clements, 2003).

Finally, we also appreciate the comments of James Stigler and Belinda Thompson, especially their extended discussion of false dichotomies, and the detrimental role they have played in research and practice in both education and psychology. They too see potential in the learning trajectory construct and add to the discussion by showing how learning trajectories focus on the details of both learning and teaching, undermining the "dumbbell theories" (Minsky, 1986) that false dichotomies propagate. They emphasize, for example, how the structure and focus on details learning trajectories can provide help teachers engage in more productive discussions. Like the other authors, they provide useful examples that illustrate their arguments, notably in their interesting international comparisons.

Several of the examples Stigler and Thompson provide emphasize the development of the subject-matter content (what we in the Common Core State Standards writing team call "mathematical progressions"). However, some of their other examples that simultaneously emphasize the children's understanding and learning, such as the notion that students who add the numerators and then add the denominators when adding fractions probably need more than reminders of the procedure. They may need to move back a level if they are not conceptualizing the fraction as a single number. Such simultaneous consideration of the development of mathematical content and the developmental course of cognitive growth is what we call the developmental progression component of a learning trajectory.

In their "deeper look" into learning trajectories, Stigler and Thompson address this characteristic directly, stating that they "are not natural objects to be discovered" but "are, instead, invented cultural artifacts." We believe there is much truth there, although our theory of hierarchic interactionalism does not dichotomize these two sources. Rather, we believe that natural progressions of thinking do exist. Even the trajectory Stigler and Thompson call "clearly not natural" follows developmental guidelines, as our research review shows. Indeed, this is why that trajectory developed the way it did in Japan—it *was* a natural developmental progression (even if it did not emerge on that basis explicitly), and was therefore particularly successful. However, we also agree that learning trajectories must be educationally engineered. In our theory, developmental progressions are born from cognitive psychology, and thus they

> play a special role in children's cognition and learning because they define sequences of levels of thinking for a specific content domain, each of which is built upon children's general intuitive knowledge and cognitive processes as they apply to the domain, as well as their previously developed patterns of thinking and learning within that domain. That is, although influenced by educational experiences and broader cultural factors, there are reliable sequences of levels of thinking, defined by identifiable patterns of mental processes and conceptual objects (actions-on-objects), through which most children progress as they learn about and gain competence. These patterns of thinking are natural to the extent that the extant biological and cognitive affordances (e.g., innate competencies) and constraints (e.g., information-processing limits and other "guidelines" described by cognitive and developmental psychology) privilege certain conceptual structures and solution strategies at certain developmental/cognitive levels and connections between contiguous levels privilege the developmental progression defined as the sequence of those levels. (Sarama & Clements, in press)

However, as Stigler and Thompson point out, learning trajectories also must be engineered. Several different curricular sequences may follow the guidelines set down by such "natural" patterns of thinking and some may be more mathematically coherent and productive than others. Thus, the demands of the subject are honored equally when constructing one or more complete developmental progression. Similarly, the third component of learning trajectories, the instructional tasks, must be invented. In our theory,

> Based on the hypothesized, specific, mental constructions (mental actions-on-objects) and patterns of thinking that constitute children's thinking, curriculum developers design instructional tasks that include external objects and actions that mirror the hypothesized mathematical activity of children as closely as possible. These tasks are sequenced, with each corresponding to a level of the developmental progressions, to complete the hypothesized learning trajectory. Such tasks will theoretically constitute a particularly efficacious educational program; however, there is no implication that the task sequence is the only path for learning and teaching; only that it is hypothesized to be one fecund route. Indeed, moving from early childhood to adulthood, experience-expectant processes provide relatively fewer guides and the role of content concerns increases, but knowledge of human development and information processing contribute to the creation of effective learning trajectories at all ages. Tasks present a problem; people's actions and strategies to solve the problem are represented and discussed; reflection on whether the problem is solved, or partially solved, leads to new understandings (mental actions and objects, organized into strategies and structures) and actions. Specific learning trajectories are the main bridge that connects the "grand theory" of hierarchic interactionalism to particular theories and educational practice. (Sarama & Clements, in press)

Thus, learning trajectories have two parents, cognitive and educational psychology and educational engineering. Thus born, the initial learning trajectory is not a finished product. It must be refined based on an understanding of multiple phases of research and development (Clements, 2007) and incorporated into the local culture. The work must involve teams with varying expertise, as Stigler and Thompson state. Ideally, different learning trajectories should be developed and evaluated using such a research framework, and ultimately compared, integrated, and scaled up. We believe this consistent with Stigler and Thompson's position. Again, they extend this discussion with the point that technology allows more extensive and intense sharing, revision, and dissemination that previously possible.

In conclusion, we thank all the reactors for their thoughtful and valuable chapters. We are honored they would take the time to respond to our chapter. However, they all contribute much more than simple critiques: Each chapter extends our brief research review, with each chapter complementing the others.

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