SYLLABUS FOR ALGEBRA PRELIMINARY EXAM

ALGEBRA GROUP
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DENVER
EFFECTIVE 2011

The exam consists of three sections: introduction to abstract algebra, group theory, and rings and fields. There will be 4 problems in every section. All 4 problems in the introduction and best 6 out of the 8 remaining problems will be graded. Each graded, perfectly solved problem counts as 10% of the final score. A score of 70% or higher guarantees a pass.

The student who wishes to take the preliminary exam in algebra should have a strong command of the topics listed on this document. For each topic, the student is expected to have a clear and rigorous understanding of the definitions, concepts and proofs, as well as related applications.

Most topics on this syllabus are covered in the courses Discrete Mathematics, Introduction to Abstract Algebra, Group Theory, and Rings and Fields, all of which are offered every academic year.

1. Introduction to Abstract Algebra

1.1. References.

- R. J. Bond and W. J. Keane, *An introduction to abstract mathematics*
- C. Pinter, *A book of abstract algebra*
- W. K. Nicholson, *Introduction to abstract algebra*
- J. J. Rotman, *A first course in abstract algebra*

1.2. Topics.

Operations on sets, groups: sets, operations, basic laws, neutral element, inverses, groups, varieties between groupoids and groups, examples

Subgroups and generators: subgroup criterion, lattice of subgroups, generators, Cayley diagrams, center

Homomorphisms and isomorphisms: morphisms, equivalence classes, kernel, examples

Euclid's division algorithm: the algorithm, generators of cyclic groups, automorphism groups of cyclic groups

Groups of permutations: S_n, cycles, decomposition of permutations, A_n, regular representation

Cyclic groups and direct products: subgroups of cyclic groups, direct products

Cosets, normal subgroups and factor groups: cosets, Lagrange's theorem, groups of prime order, normal subgroups, normality of center and kernels, factor group

Fundamental homomorphism theorem, isomorphism theorems: FHT, inner automorphisms, applications, the 2nd and 3rd isomorphism theorems

Cauchy's theorem: Cauchy's theorem, classification of very small groups
Rings and fields: rings, domains, integral domains, division rings, fields, Wedderburn theorem, constructions, properties of invertible elements
Subrings, ideals and homomorphisms: subring, ideal, ideals of $\mathbb{Z}$, PIDs, FHT for rings, factor rings, prime and maximal ideals, applications
Integral domains and fields of quotients: characteristic, field of quotients
Rings of polynomials, factorization in integral domains: polynomials, division algorithm, ideals in polynomial rings, prime and irreducible elements, associates, UFDs, primes and irreducibles in UFDs, norm, examples
Finite fields: constructions, existence

2. Group Theory

2.1. References.

- J. J. Rotman, Group Theory
- D. J. S. Robinson, A course in the Theory of Groups
- T. W. Hungerford, Algebra

2.2. Topics.

Permutation groups: S_n, A_n, simplicity of A_n
Group actions: group actions, orbits, stabilizers, examples
Abelian groups: fundamental theorem of finitely generated abelian groups
Sylow theorems: p-groups, Sylow theorems, applications
Subnormal series: subnormal series, central series, nilpotent groups, commutators, solvable groups
Extensions: semidirect product, central extensions, Schreier extensions
Matrix and classical groups: matrix groups, classical groups and their simplicity
Torsion and divisibility: torsion and torsion-free groups, divisible and uniquely divisible groups
Free groups: free groups, generators and presentations, the word problem

3. Rings and Fields

3.1. References.

- C. Lanski, Concepts in abstract algebra
- T. W. Hungerford, Algebra

3.2. Topics.

PIDs, UFDs, Euclidean domains: PIDs, UFDs and Euclidean domains
Zorn's lemma: Zorn's lemma, axiom of choice, applications
Chain conditions: ascending and descending chain conditions of ideals, Noetherian rings, Hilbert basis theorem, Artinian rings
Field extensions: extensions of fields, integral extensions, algebraic extensions, split extensions, transcendental extensions
Intro to Galois theory: subfields, solvable groups, Galois correspondence, applications, classical problems
Finite fields: existence and uniqueness of finite fields, trace, norm