SYLLABUS FOR ANALYSIS PRELIMINARY EXAM

ANALYSIS GROUP
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF DENVER
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The exam consists of three sections: real analysis, metric space theory and topology. As a general guideline, the exam is expected to be organized with 40% real analysis, 30% metric spaces and 30% topology, with the intent that a score of 60% or higher guarantees a pass.

The student who wishes to take the preliminary exam in analysis should have a strong command of the topics listed on this document. For each topic, the student is expected to have a clear and rigorous understanding of the definitions and proofs, as well as several applications in mind, and to have practiced many problems on the matter.

The list of topics on this syllabus should be covered in the classes of calculus 1, 2, 3, real analysis, metric spaces and topology. However, if some topics were not covered in class, it is the responsibility of the student to study them for the preliminary exams.

1. Real Analysis

1.1. References.

- W. Rudin, *Introduction to real analysis*, Academic Press.
- S. Abbott, *Understanding Analysis*, Springer.

Note: This section includes all the knowledge of calculus. Please refer to your calculus textbook if necessary.

1.2. List of topics.

The set of real numbers: Supremum and infimum, Archimedean property.

Sequences in $\mathbb{R}$: Convergence of sequences, algebraic and order properties of limits, Cauchy sequences, completeness of $\mathbb{R}$, monotone sequences, monotone subsequence theorem, monotone convergence theorem, $\liminf$, $\limsup$.

Topology of $\mathbb{R}$: Open and closed sets, limit points, closure of a set, compact sets, characterization of compactness as closed and bounded in $\mathbb{R}$, connected sets are intervals in $\mathbb{R}$. Limit of a function at a point.

Continuous functions on $\mathbb{R}$: Continuity at a point, continuity on a set, equivalence between continuity and sequential continuity, continuous image of a compact (extreme value theorem), continuous image of a connected set (intermediate value theorem), uniform continuity, continuous functions on compact sets are uniformly continuous.

Differential Calculus: Derivative of a function, algebraic properties of derivatives, Rolle's theorem, mean value theorem and applications, Taylor polynomials and Taylor's theorem with remainder.

Integral Calculus: Riemann integral, characterization of Riemann integrable functions as bounded and continuous almost everywhere, fundamental theorem of calculus.

Series: Convergence of series, absolute convergence, basic comparison theorems, power series, analytic functions and Taylor series.

2. Metric Space Theory

2.1. References.

- W. Rudin, *Introduction to real analysis*, Mc Graw-Hill.
- K. Stromberg, *Introduction to Classical Real Analysis*, Wadsworth and Brooks.
- H.L. Royden, *Real Analysis*, Prentice Hall.

2.2. List of Topics.

Metric spaces: Distance on a set, metric space, function limits, continuous function, uniformly continuous function, Lipschitz functions.

Basic Topology of metric spaces: Open and closed balls, open sets, closed sets.

Completeness: Cauchy sequences, completeness of metric spaces, contraction mapping theorem.

Compactness: Compact metric space, uniform continuity and compactness, continuous image of a compact set, compact subsets of $\mathbb{R}$, compact metric spaces are complete.

Baire category: G_δ and F_σ sets, Baire category theorem.

Uniform convergence: Uniform convergence, relation to pointwise convergence, limit of uniformly convergent sequence of continuous functions is continuous, interchange of limits theorems.

Connectedness: Connected spaces, continuous image of a connected set, connected subsets of $\mathbb{R}$ are intervals.

3. Topology

3.1. References.

- J. Munkres, *Topology*, Prentice Hall.
- S. Willard, *General Topology*, Dover.

3.2. List of topics.

Topological spaces: Topology on a set, topological space, continuous function on a topological space, open sets, closed sets, neighborhood of a point, topology generated by a collection of subsets, topology generated by a family of functions. Trace topology.

Bases: Basis and subbasis for a topology, topology generated by a basis, characterization of continuity in terms of basis, basis of neighborhoods.

Closure, Interior: Definitions of closure and interior of a set in a topological space(respectively smallest closed super-set and largest open subset), limit points, characterization of closures, dense subsets, characterizations of continuity using closures.

Fundamental examples: Initial topologies, final topologies (including quotient), metric space topology, order topology, product topologies.

Limits: Separation properties: T0, T1, T2 (Hausdorff), Definition of the limit of a function at a point, basic properties, continuity at a point, relation with continuity on a set. Limit of sequences.

Compactness: Compact spaces, compact metric spaces, Bolzanno-Weierstrass and Heine-Borel compactness are equivalent for metric spaces, continuous image of a compact set, compact subsets of $\mathbb{R}$, uniform continuity, Heine theorem (continuous functions on compact sets are uniformly continuous), Tychonoff theorem.

Connectedness: Connected spaces, continuous image of a connected set, connected subsets of $\mathbb{R}$ are intervals.